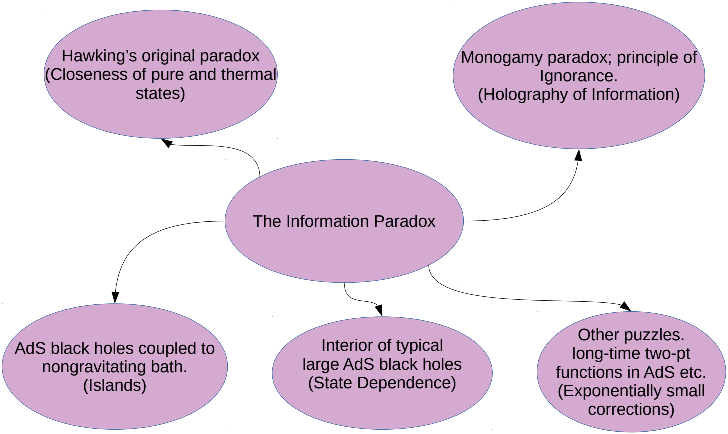
Summary

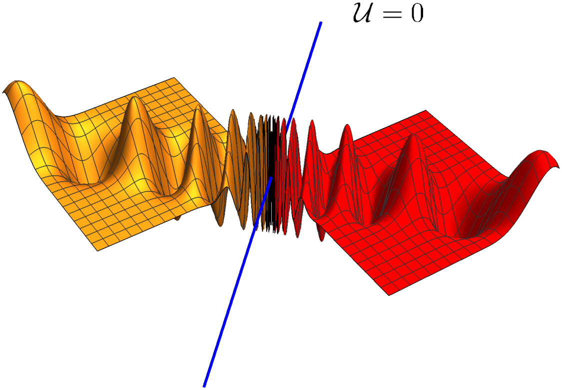
April 15, 2021

The Information Paradox



The information paradox is a web of interconnected puzzles that teach us lessons about quantum gravity.

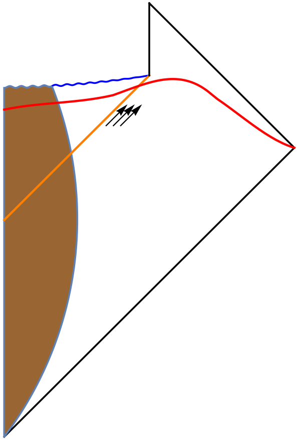
Entangled modes across a null surface



$$(\mathfrak{a} - e^{-\pi\omega_0}\widetilde{\mathfrak{a}}^\dagger)|\Psi\rangle = 0, \qquad (\mathfrak{a}^\dagger - e^{\pi\omega_0}\widetilde{\mathfrak{a}})|\Psi\rangle = 0.$$

(Lectures 1 – 2)

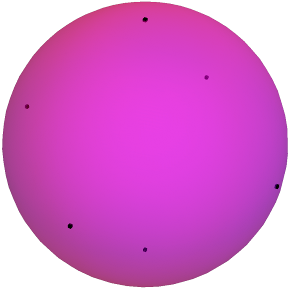
Hawking radiation and Hawking's original paradox



$$\langle \Psi | a_{\omega,\ell} a^{\dagger}_{\omega,\ell} | \Psi \rangle = \frac{e^{-\beta\omega}}{1 - e^{-\beta\omega}}$$

This leads to a robust derivation of Hawking radiation and Hawking's original paradox. (Lectures 3 – 5)

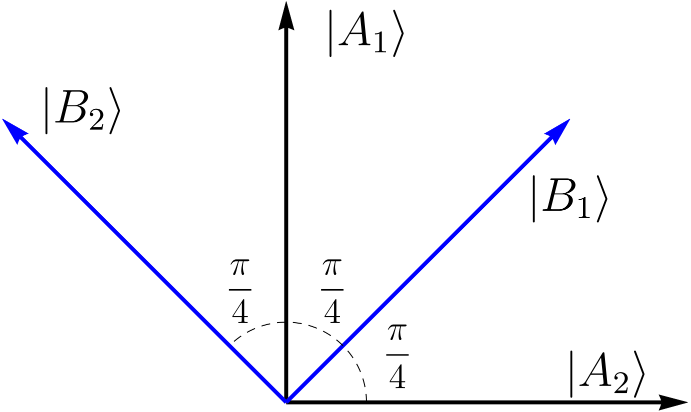
Thermalization



$$\int d\mu_\Psi \left(\mathrm{Tr}(\rho_E P) - \langle\Psi|P|\Psi\rangle\right)^2 \leq \frac{1}{(W+1)}.$$

Pure states are exponentially close to mixed states, which resolves the simplest version of the paradox. (Lectures 6 – 7)

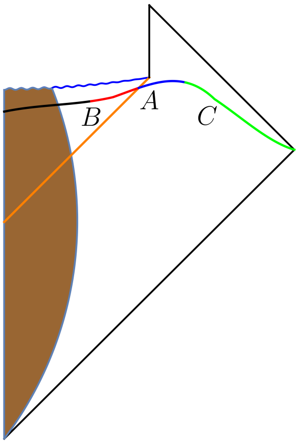
Results from quantum information



$|\langle C_{AB}\rangle| \leq 2\sqrt{2}$; $\quad \langle C_{AB}\rangle^2 + \langle C_{AC}\rangle^2 \leq 8$.

(Lecture 8)

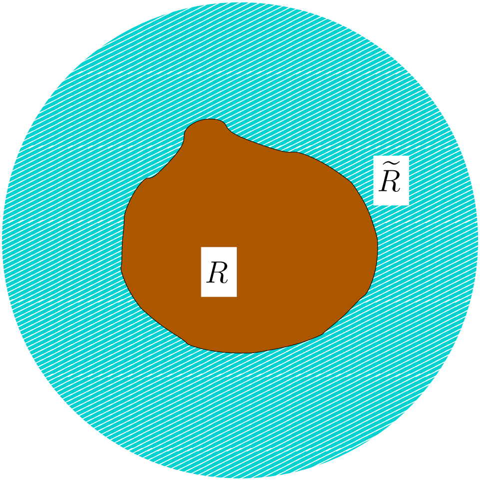
A monogamy paradox



$$\langle C_{AB} \rangle^2 + \langle C_{AC} \rangle^2 = 8 + \frac{4}{(1+e^{-\beta\omega})^2}\left(1 + 6e^{-\beta\omega} + e^{-2\beta\omega}\right)?$$

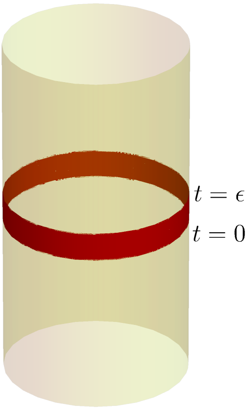
This was used to argue for the presence of structure at the black hole horizon. (Lectures 9–10)

Holography of information



In a theory of quantum gravity, information available on a Cauchy slice is also available near the boundary of the slice.
(Lecture 10)

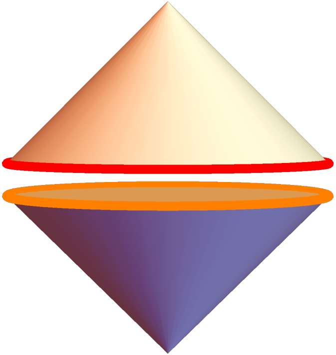
Holography of information in AdS



$$A = \sum_{n,m} c_{nm} X_n P_0 X_m^\dagger$$

In asymptotically AdS spacetimes, all information is available in a time band of extent ϵ on the boundary. (Lectures 11–12)

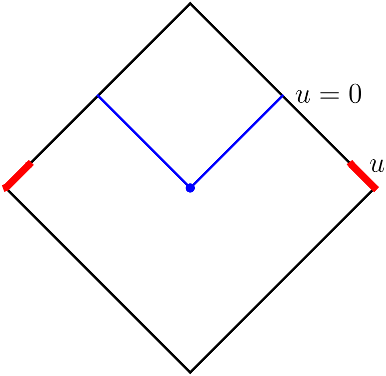
Holography of information in flat space



$$A = \sum c(n, m, s, s') X_n T_{\{s\},\{s'\}} X_m^\dagger.$$

In 4d asymptotically flat spacetimes, information about massless particles is available near $\mathcal{I}^+_-$. (Lectures 13–14)

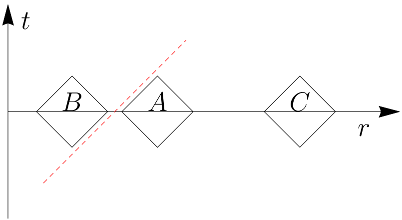
Low energy tests of holography of information



$$\langle f|M(-\infty)O(u,\Omega')|f\rangle = G\lambda \int_0^1 \frac{f(x,\Omega')}{(x-u-i\epsilon^+)}dx$$

The holography of information is visible within low-energy physics. (Lectures 13, 15)

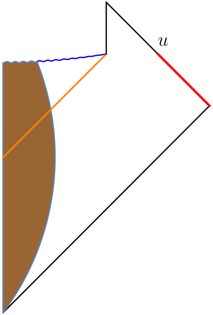
A perspective on black hole information

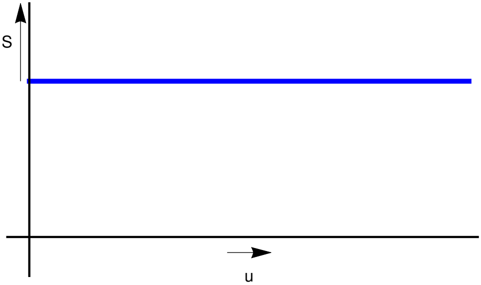


$$\langle 0|C_{AB}|0\rangle^2 + \langle 0|C_{AC}|0\rangle^2 > 8?$$

Information about the black hole interior is always available outside. Ignoring this redundancy can be shown to lead to a monogamy paradox. (Lecture 15)

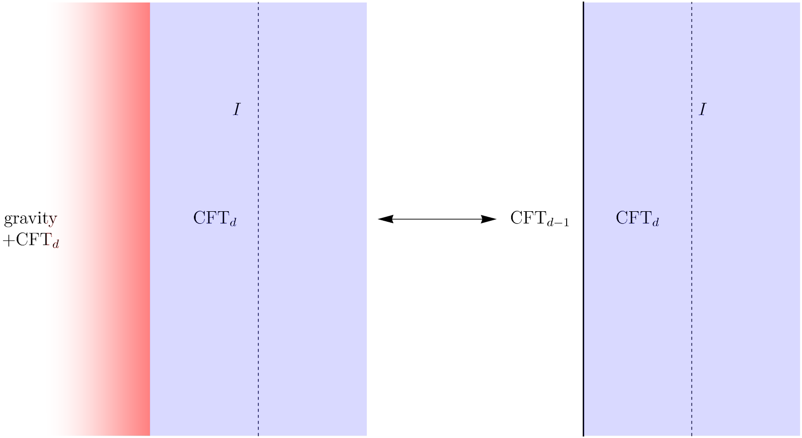
von Neumann entropy at $\mathcal{I}^+$





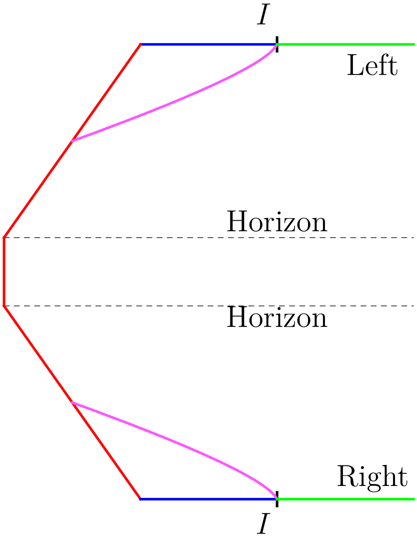
The fine-grained von Neumann entropy of $(-\infty, u)$ of $\mathcal{I}^+$ in gravity is independent of u! (Lecture 16)

AdS black holes and a nongravitational bath



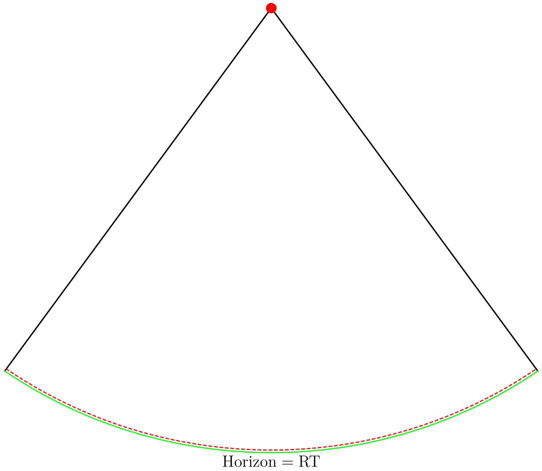
A naive holographic computation of the entropy of the bath would suggest an ever-increasing entropy. (Lecture 18)

Islands



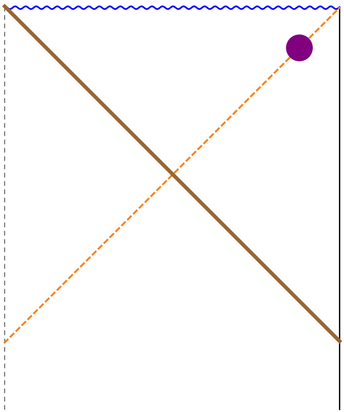
The paradox is resolved by a phase transition between RT surfaces. (Lecture 19)

Gravity in the bath



Introducing gravity in the bath leads to a constant Page curve, as in flat space. (Lecture 20)

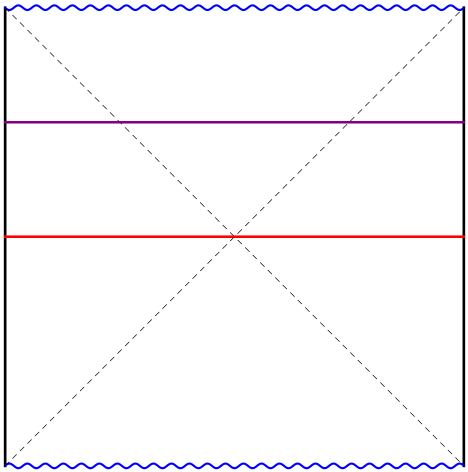
Large AdS black holes



$$\frac{1}{Z(\beta)}\mathrm{Tr}(e^{-\beta H}\widetilde{a}_{\omega,\ell}\widetilde{a}^{\dagger}_{\omega,\ell}) = -\frac{e^{-\beta\omega}}{1-e^{-\beta\omega}},$$

Large AdS black holes dominate the microcanonical ensemble.
This leads to new paradoxes. (Lecture 21)

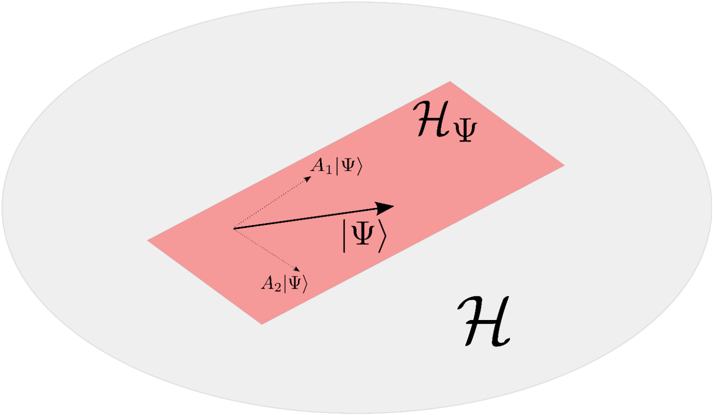
The eternal black hole



$$|\Psi_\tau\rangle = e^{-i(H_L+H_R)\tau/2}|\Psi_{\rm tfd}\rangle.$$

These paradoxes can also be extended to the eternal black hole. (Lecture 22)

Interior reconstruction

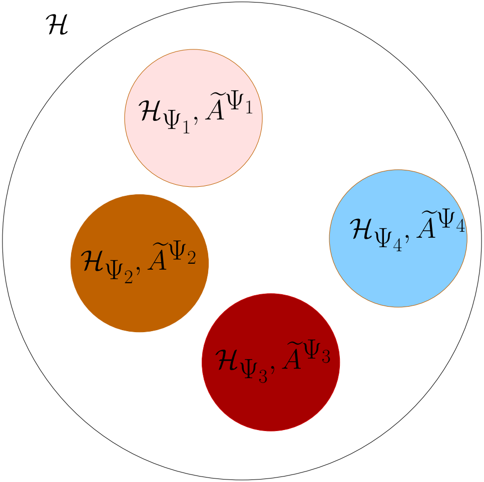


Mirror operators reconstruct the interior about a smooth microstate

$$\widetilde{A}_n A_m |\Psi\rangle = A_m e^{-\frac{\beta H}{2}} A_n^\dagger e^{\frac{\beta H}{2}} |\Psi\rangle.$$

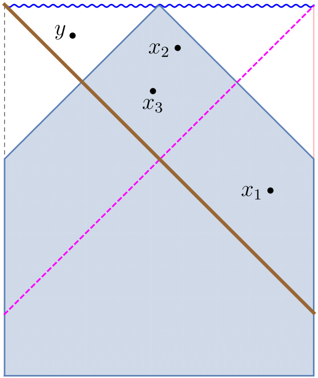
(Lecture 23)

State dependence



If the mirrors are allowed to be state dependent, even typical states are smooth. (Lecture 24)

Consistency of state dependence



$$\left|\langle\Psi|U^{\dagger}AU|\Psi\rangle - \langle\Psi|A|\Psi\rangle\right| \leq 2\sqrt{\beta\delta E}\sigma$$

State dependence suggests black holes are unusually sensitive to low-energy excitations. Refining the notion of "simple" observables removes this anomaly. (Lecture 25)

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