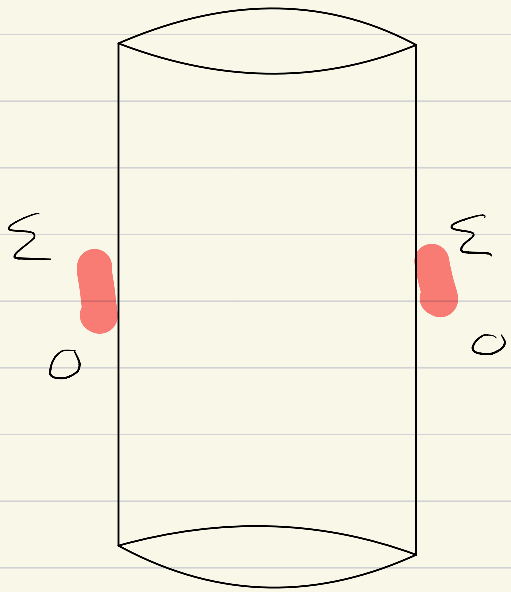
24-2-2021

Lecture 13: Low energy tests of PHoI in AdS and flat space background

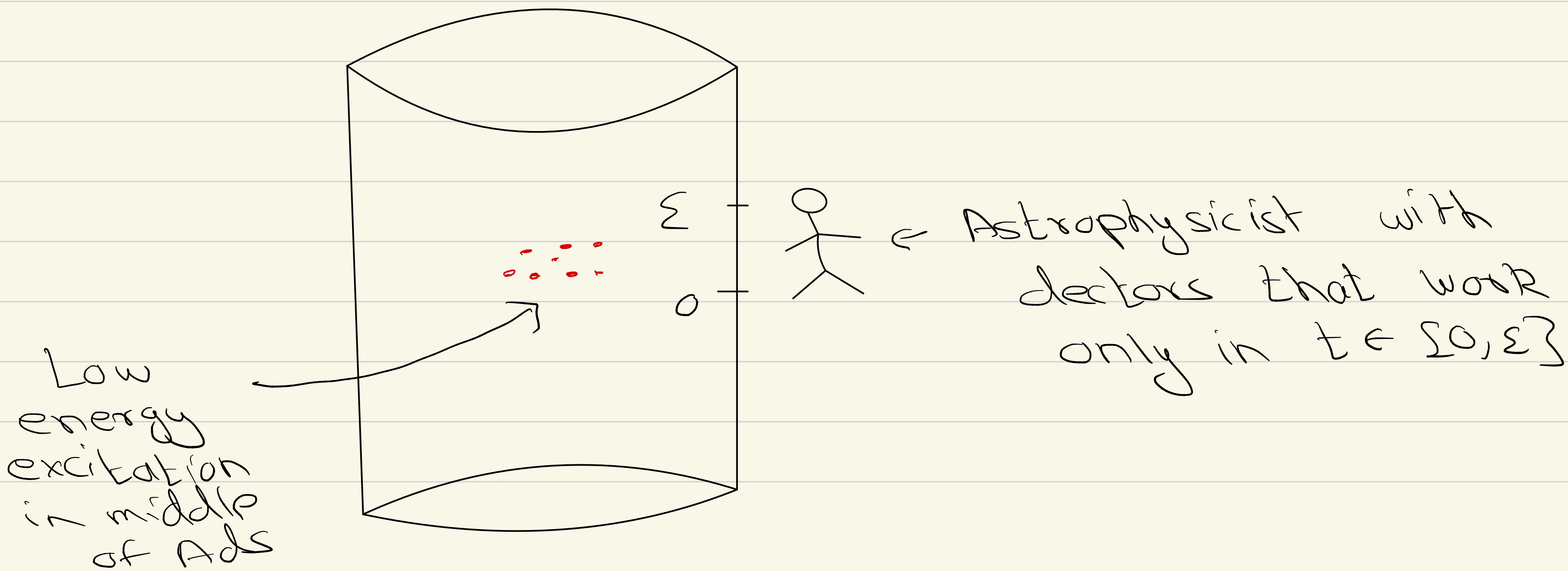
In the last lecture we proved that, in AdS, all operators in the theory could be represented by operators in the time band $[0,\epsilon]$



This might have seemed like an abstract operator-theoretic result.

Now we want to set up a thought experiment using **only low-energy physics** to test this principle.

The idea is as follows



Task:

The bulk is in a state $|g\rangle$ and the observers need to identify it.

The state may have some components of arbitrarily high energy but the observers are told that

$$1 - \left| P_{E<\Lambda} |g\rangle \right|^2 \ll 1 .$$

so, most of the components of the state are below Λ.

[This is important since even in LQFT localized states have small high-energy components.]

The observers need to find this "low energy" part.

Abilities:

The observers are given the abilities of standard Q.M. experiments

a) If X is a **simple** low-energy operator the observers can act with the unitary

$$e^{iJX}$$

for small J.

b) They can measure the energy

c) All detectors work only between $t \in [0, \varepsilon]$.

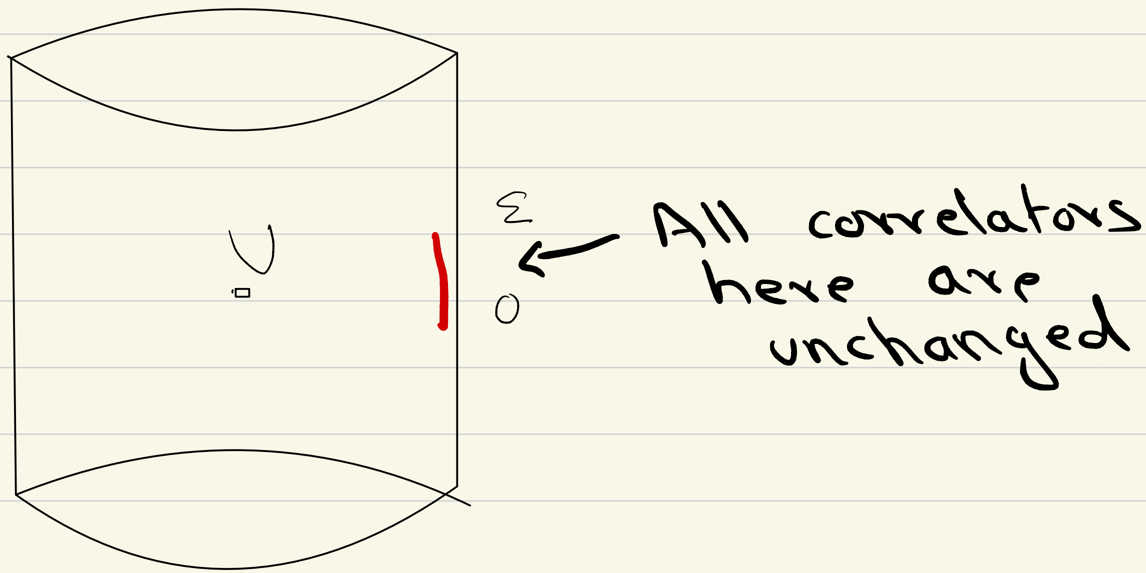
Warmup Task 1

Lets start with the simplest test.

a) Determine if $|g\rangle = |0\rangle$ or not.

Note this is impossible in a theory without gravity

In a LQFT, the observers can never distinguish $|0\rangle$ from $U|0\rangle$ where U is a bulk unitary



In gravity, this can be done very easily

Measure the energy and determine the probability of getting 0.

By the Born rule, this is

$$\langle g | P_0 | g \rangle = |\langle 0 | g \rangle|^2$$

If $|g\rangle = |0\rangle \Leftrightarrow \langle g | P_0 | g \rangle = 1$

success in gravity!

Lets do a second more non-trivial state.

Warmup Task 2:

Let X be a simple, Hermitian operator near the boundary and let

$$|X\rangle = X|0\rangle$$

Say the observers have determined that

$$\langle g|0\rangle = 0$$

and are now asked to find out

is $|g\rangle = X$ or not?

Note that now $|X\rangle$ is not determined by a conserved charge.

We now follow a 2-step procedure.

1) Act with e^{iJx}

2) Measure the energy at $O(J^2)$ and determine the prob that it is 0.

Let us compute the effect of this manipulation

We start with $|g\rangle$ and step 1 results in

$$|g\rangle \rightarrow e^{iJx} |g\rangle$$

So the answer for step 2 is

$$\langle g | e^{-iJx} P_0 \, e^{iJx} | g \rangle$$

Let us expand this to $O(J^2)$

$$\langle g | \left(1 - iJx - \frac{J^2 x^2}{2}\right) |0\rangle\langle 0| \, 1 + iJx + \frac{J^2 x^2}{2} \, |g\rangle$$

Recall that $\langle g | 0 \rangle = 0$ by assumption

so to $O(J^2)$ we have **only 1 term!**

$$i(-i) \; \langle g | J x | 0 \rangle \cdot \langle 0 | J x | g \rangle + O(J^3).$$

Final answer is $J^2 \langle g | x \rangle \langle x | g \rangle = J^2 |\langle g | x \rangle|^2$

So we again find that, by this two-step procedure the observers can determine if the state in $|X\rangle$ or not.

As usual, in a LQFT

$$|X\rangle \quad \text{and} \quad U|X\rangle$$

cannot be distinguished.

Now we are almost done as regarding the original task.

Note that when X ranges over low energy boundary hermitian operators $|X\rangle$ ranges over a basis of low-energy states

Note that we cannot get all low-energy states due to the restriction that "X" be Hermitian.

eg

$(X_1 + iX_2)|0\rangle$ can produce $|X_1\rangle + i|X_2\rangle$

but this is not Hermitian if X_1, X_2 are.

By acting with a preliminary unitary, we can take

$$|g\rangle \to U|g\rangle$$

so that

$$\langle 0|U|g\rangle = 0$$

This is very easy. [Rotating a single local degree of freedom can make the state orthogonal to the vacuum.]

Let us assume this has been done and let us use the notation $|g\rangle$ for the new state.

Then "warmup task 2" allows us to determine

$$|\langle g|x\rangle|^2$$

for $|x\rangle$ ranging over a basis.

To complete the task, we need to determine the phase $\langle g|x\rangle$

This can be done as follows. Pick your favourite operator, X_r, and declare that

$$\langle g|X_r\rangle \text{ is real.}$$

We can always do this because the overall phase in $|g\rangle$ is arbitrary.

"warmup task 2" also allows us to determine

$$\left| \langle g | x \rangle + \langle g | x_\gamma \rangle \right|^2$$

where X is Hermitian.

Independently determinable

$$\left| \langle g|x\rangle + \langle g|x_\gamma\rangle \right|^2 = \left|\langle g|x\rangle\right|^2 + \left|\langle g|x_\gamma\rangle\right|^2 + 2\langle g|x_\gamma\rangle \,\mathrm{Re}(\langle g|x\rangle)$$

determinable by acting with $e^{iJ(x+x_\gamma)}$ and then measuring energy

known

can be found since all else in eqn is known

This leaves us with a sign ambiguity in $\mathrm{Im}\langle g|x\rangle$ which can be fixed with more work.
see arXiv: 2008.01740

The punch-line is simple.

The fact that bulk information on a Cauchy slice is also available near the boundary is not some abstract statement, but can be verified concretely in low-energy effective field theory.

Flat space preliminaries

We now turn to flat space. Let us consider some geometric preliminaries.

We are interested in spacetimes that, at infinity, tend to 4d Minkowski space.

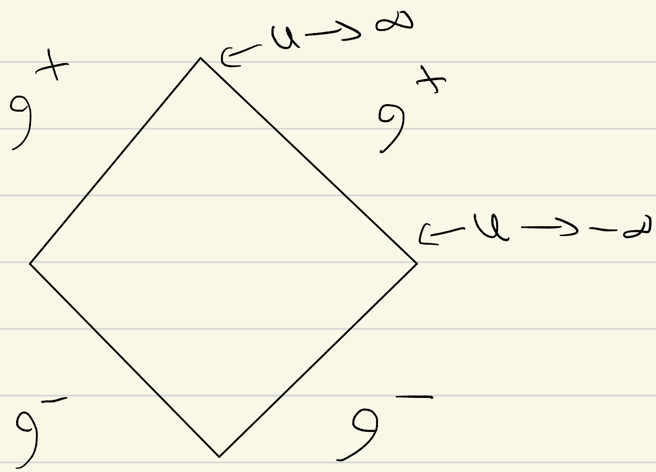
This means that

$$ds^2 \xrightarrow[r\to\infty]{} -du^2 - 2du\,dr + r^2 \gamma_{AB} d\Omega^A d\Omega^B + (\text{subleading})$$

[Note we have restricted to 4d for reasons that will become clear
γ_{AB} is the round metric on the sphere]

Before we discuss the subleading term, we make some remarks about the Penrose diagram and leading term.

1) This line-element parametrizes $\mathscr{I}^+$



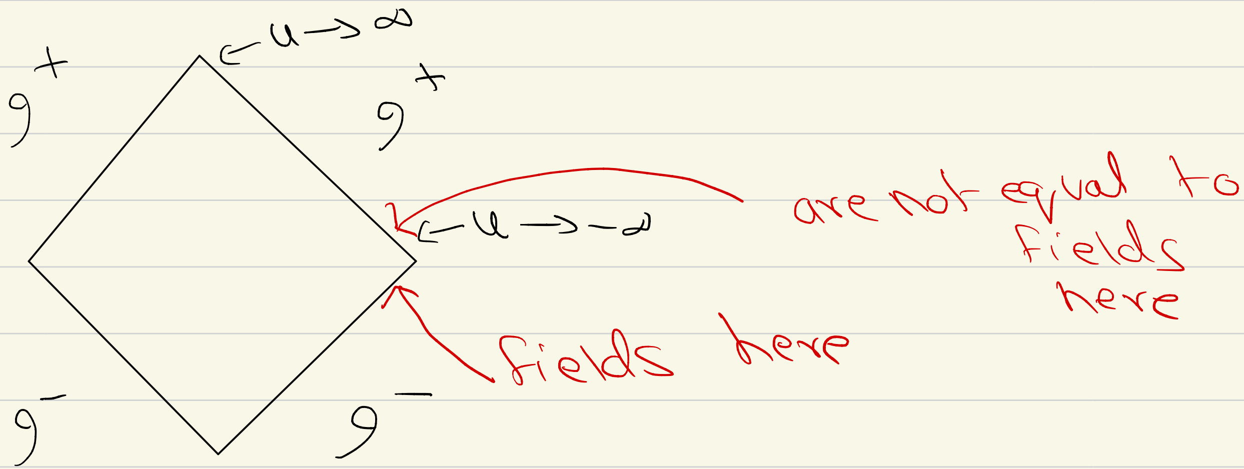
2) The past of future null infinity is $\mathscr{I}^+_-$ at $u \to -\infty$

3) Note we can also talk about the future of past null infinity $\mathscr{I}^-_+$

$$\mathscr{I}^+_- \neq \mathscr{I}^-_+$$

Although spatial infinity, i^0 looks like a "point" on the Penrose diagram, that is deceptive

[See Strominger: 1703.05448]



4) In the G.R. literature it is common to discuss black holes separately and not as part of this diagram.

But for us, black holes always evaporate

So the Penrose diagram is ultimately trivial even if the bulk is very complicated.

We now need to set boundary conditions on the allowed fluctuations

These are most easily specified in "Bondi gauge" we set

$$g_{rr} = g_{rA} = 0$$

and also $\quad \partial_r \det\left(\frac{g_{AB}}{r^2}\right) = 0$

In this gauge:

$$g_{uu} \longrightarrow -1 + O\left(\frac{1}{r}\right)$$

$$g_{ur} \longrightarrow -1 + O\left(\frac{1}{r^2}\right)$$

$$g_{uA} \longrightarrow O(1)$$

$$g_{AB} \longrightarrow r^2 \gamma_{AB} + O(r)$$

One also demands some conditions on the Weyl tensor

[See Compere 1801.07064
and Strominger 1703.05448 (and also exercise 10)]

for this relation

We then find

$$ds^2 = -du^2 - 2du\,dr + r^2 \gamma_{AB}\, d\Omega^A d\Omega^B$$
$$+ \frac{2m}{r} du^2 + r\, C_{AB} d\Omega^A d\Omega^B + D^B C_{AB}\, du\, d\Omega^A$$

Note C_{AB}, m are fns of u and Ω^A

Here

C_{AB} is called the "shear"

a) It must be symmetric [since it contracts $dx^A dx^B$

b) It satisfies $\gamma^{AB} C_{AB} = 0$

so C_{AB} has 2-independent components and these contain info about the two dynamical graviton components at $\mathscr{I}^+$.

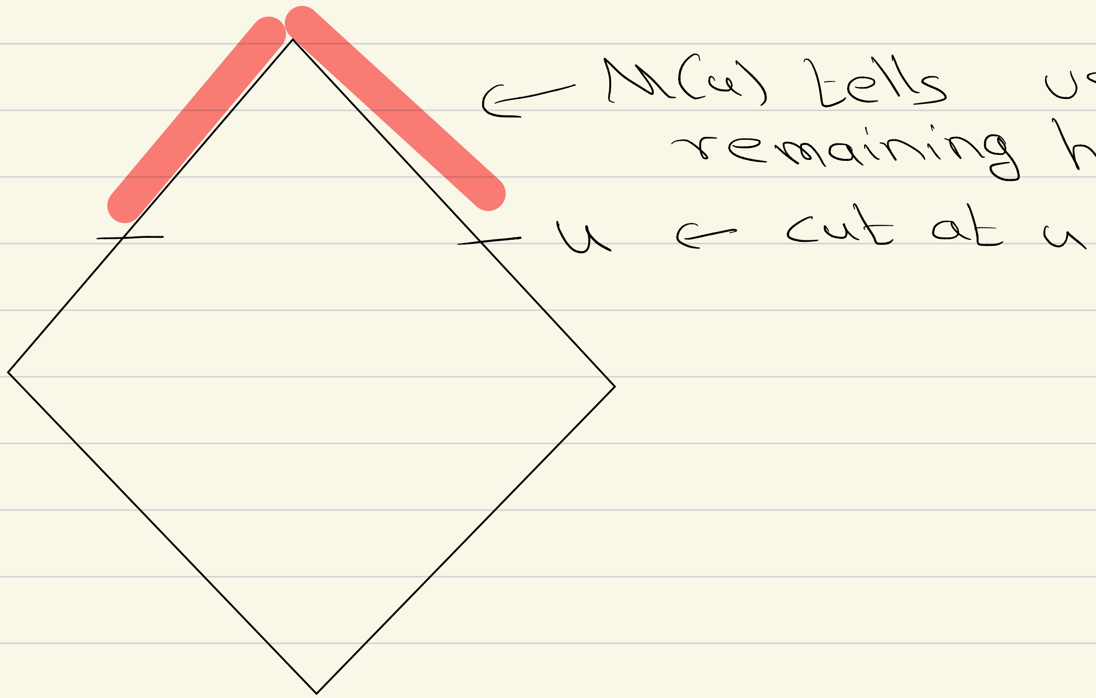
The Bondi "news" is defined by

$$N_{AB} = \partial_u C_{AB}$$

m is called the Bondi mass aspect.

The integral of the mass aspect is the Bondi mass

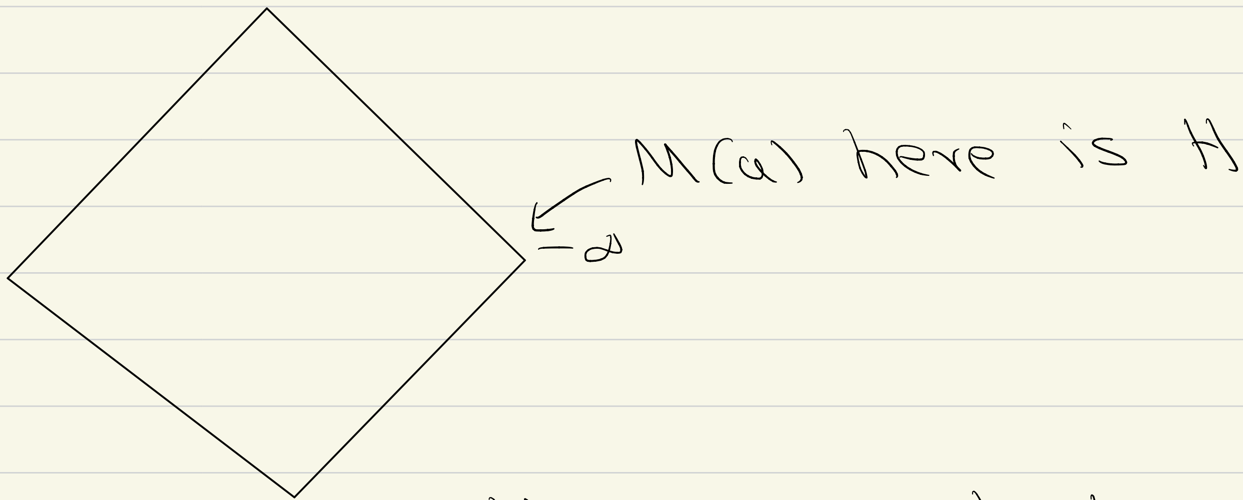
$$M(u) = \int \sqrt{\gamma}\, m_B(u, \Omega)\, d^2\Omega$$



The limit

$$\lim_{u \to -\infty} M(u) = H$$

is the canonical Hamiltonian.



We will later discuss the $u \to -\infty$ limit of the mass aspect $m(u, \Omega^A)$

Apart from the metric, we may have other dynamical fields in the theory.

For instance if there is a scalar field in the theory, we demand

$$\Phi(r, u, \Omega) \xrightarrow[r \to \infty]{} \frac{1}{r} O(u, \Omega)$$

[For gauge fields, in Lorentz gauge $\nabla_\mu A^\mu = 0$ we have the components $A_A(u, \Omega)$

sphere components

See exercise 2 of 1703.05448]

In any realistic theory, we will also have massive fields.

But massive fields fall off exponentially in r near $\mathcal{I}^+$.

They come out at future timelike infinity i^+. We will not say much about them.

This is not a huge omission since, for a large black hole, recall $T \sim \frac{1}{r}$

So if a black hole is very large to start with, most of its radiation is in terms of massless particles.