3 March 2021 Lecture 15

Low energy tests of HoI in flat space

We again want to step away from formal algebraic considerations and explore the low-energy manifestation of the principle of HoI in flat space

Let $|0\rangle$ be some vacuum (possibly a superposition of several soft vacua.)

Let $O(u, \Omega)$ be the boundary value of a scalar field.

Consider

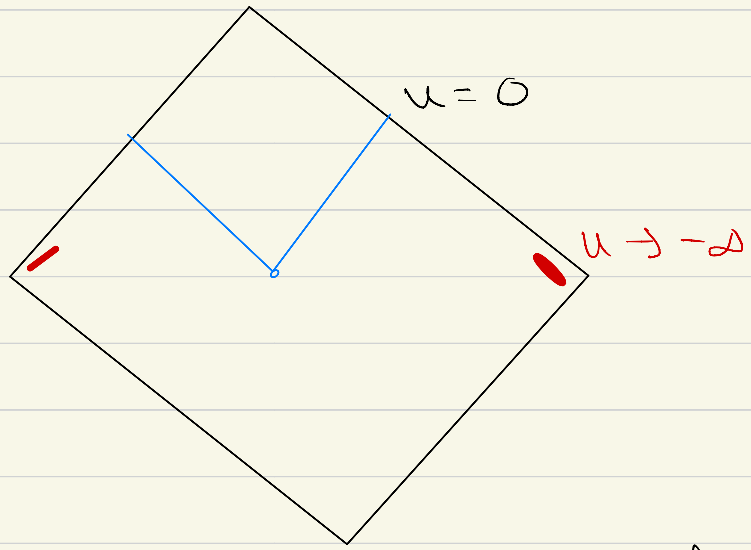
$$|f\rangle = e^{-i\lambda \int f(u,\Omega)\, O(u,\Omega)\, du\, d\Omega} |0\rangle$$

We take $f(u,\Omega)$ to have **compact support** in $u \in [0,1]$

Note this is a very simple sort of state.

Here λ is a perturbative parameter and we assume we have the ability to make observations at different small values of λ.

Physically, one way to think of this excitation as follows.



We would like to consider whether observations near $u \to -\infty$ can be used to detect the form of $f(u, \Omega)$.

Obviously, this is impossible in a LQFT.

But in gravity this can be done as follows:

Consider the correlator

Note different u coordinates

$$\lim_{\tilde{u} \to -\infty} \langle f | M(\tilde{u}) \; O(u, \Omega') | f \rangle$$

$$= 4\pi G \langle f | H \; O(u, \Omega') | f \rangle$$

where $u \in (-\infty, -\frac{1}{\varepsilon})$

We are just considering the two-point correlator of a metric fluctuation and a matter field.

Let us work out the correlator to first order in λ [Recall we are allowed to explore the

We need to compute

$$\langle 0 | e^{-i\lambda \int F(u',\Omega')\, O(u',\Omega')\, du'\, d\Omega'} H \; O(u,\Omega)\, e^{i\lambda \int F(u'',\Omega'')\, O(u'',\Omega'')\, du''\, d\Omega''} |0\rangle$$

At first order in λ there is

$$\int \Big[-i\lambda \; \langle 0 | O(u',\Omega')\, H \; O(u,\Omega) | 0 \rangle$$

$$+ i\lambda \; \langle 0 | H \; O(u,\Omega)\; O(u',\Omega') | 0 \rangle \Big] F(u',\Omega')\, du'\, d\Omega'$$

But recall

$$\langle 0| H = 0$$

But also

$$O(u',\Omega') H = H\, O(u',\Omega') + i \frac{\partial}{\partial u'} O(u',\Omega')$$

This is just the Heisenberg equation of motion but it also follows from the commutators of asymptotic quantization that we described last time.

So the correlator becomes

$$\int -i\lambda \langle 0| F(u',\Omega')\, O(u',\Omega')\, H\, O(u,\Omega)|0\rangle\, du'$$

$$= \lambda \int \langle 0| F(u',\Omega')\, \partial_{u'} O(u',\Omega')\, O(u,\Omega)|0\rangle\, du'\, d\Omega'$$

We can also compute the 2-point function

$$\langle \partial_{u'} O(u', \Omega') \, O(u, \Omega) \rangle = \frac{-1}{4\pi} \frac{1}{u' - u - i\varepsilon} \delta^2(\Omega, \Omega')$$

[see assignment question]

So putting everything together

$$\lim_{\tilde{u} \to -\infty} \langle F | M(\tilde{u}) \, O(u, \Omega) | F \rangle = G \lambda \int \frac{f(x, \Omega) \, dx}{x - u - i\varepsilon} + O(\lambda^2)$$

Note even though $u \in (-\infty, \frac{-1}{\varepsilon})$ this integral runs over all x

Also note Ω is fixed.

We can use these observations to uniquely reconstruct $f(u,\Omega)$

1) Formal argument

Knowledge of $\quad C(u,\Omega) = \int \frac{f(x,\Omega)}{x-u-i\varepsilon}\, dx$

for $u \in (-\infty, \frac{-1}{\varepsilon})$

fixes $f(x,\Omega)$ uniquely.

<u>Proof:</u> Say there were two functions f_1, f_2 that yield the same function.

Then $\quad \int \frac{f_1(x,\Omega) - f_2(x,\Omega)}{x-u-i\varepsilon}\, dx = 0 \quad \forall u \in (-\infty, \frac{-1}{\varepsilon})$

But this function is analytic in the upper u-plane. So if it vanishes for $u \in (-\infty, -\frac{1}{\varepsilon})$, it must vanish everywhere.

So then $f_1 = f_2$.

2) Hands on argument

We also have

$$C(u, \Omega) = \int \frac{F(x, \Omega)\, dx}{x - u - i\varepsilon} = -\sum_{n=0}^{\infty} \frac{1}{u^{n+1}} \int x^n F(x, \Omega) dx$$

So by making measurements at different values of u, the observers can extract the **moments** of $F(x)$.

Since $f(x)$ has compact support in $[0,1]$ an $O(1)$ number of moments is sufficient to obtain an excellent approximation to $f(x)$.

[Explain]

This example also helps to illustrate various physical aspects of the principle of HoI.

a) the price of being far away
b) classical / nongravitational limit
c) global symmetries

a) the price of being far away

Since $f(x)$ has support in $[0,1]$
the ratio $\frac{x}{u}$ is small

so in

$$C(u,\Omega) = \int \frac{f(x,\Omega)\,dx}{x-u-i\varepsilon} = -\sum_{n=0}^{\infty} \frac{1}{u^{n+1}} \int x^n f(x,\Omega)\,dx$$

the higher moments are harder to extract.

b) classical / nongravitational limit:

As a consistency check, these effects should vanish in the classical / nongravitational limit:

Recall that we found that

$$\langle f | M(-\lambda) O(u, \Omega') | f \rangle = G\lambda \int \frac{F(x, \Omega)\, dx}{(x - u - i\varepsilon)} + O(\lambda^2)$$

If we restore constants we find

$$G \to \frac{\hbar G}{c^3}$$

M has dim of length [appears in metric as $\frac{M}{r}$]

O is dimensionless.

$\lambda \int F(x, \Omega)\, dx$ is dimensionless

$\frac{1}{(x - u - i\varepsilon)}$ has units L^{-1}; so G must be converted to L^3

So in the limit that $\hbar \to 0$ or $G \to 0$, this effect vanishes.

This is **expected**

Classically, we obtain information about the total energy of the excitation (at $O(\lambda^2)$) but here we are measuring

correlations of energy fluctuations with other dynamical fields.

Global symmetries

Say we have another field with boundary value $\tilde{O}$ related to O by a global symmetry

Say we consider states

$$|\tilde{f}\rangle = e^{i\int f(u,\Omega)\tilde{O}(u,\Omega)} |0\rangle$$

$$|f\rangle = e^{i\int f(u,\Omega) O(u,\Omega)} |0\rangle$$

It is easy to check

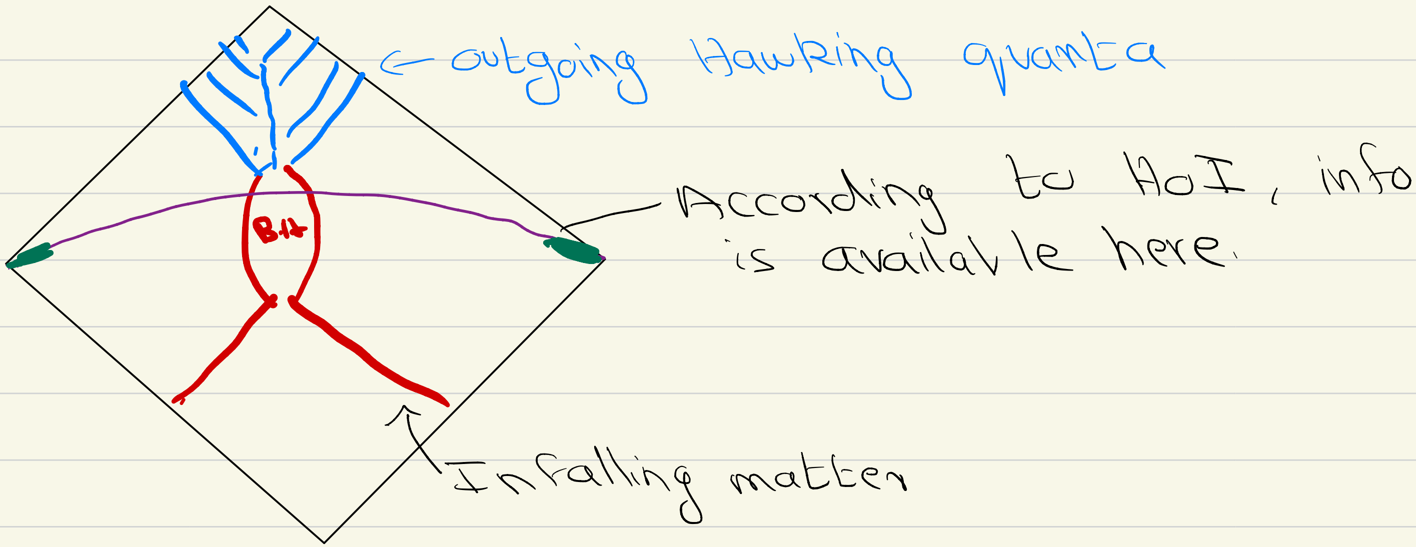
	$\langle M(-\omega) O(u,\Omega)\rangle$	$M(-\omega)\tilde{O}(u,\Omega)$
$\|f\rangle$	$C(u,\Omega)$	0
$\|\tilde{f}\rangle$	0	$C(u,\Omega)$

Perspective on Black Hole Information

Relevance for Hawking's original paradox

The principle of holography of information tells us that

"the exterior of the b.h. always retains a copy of the information in the interior"



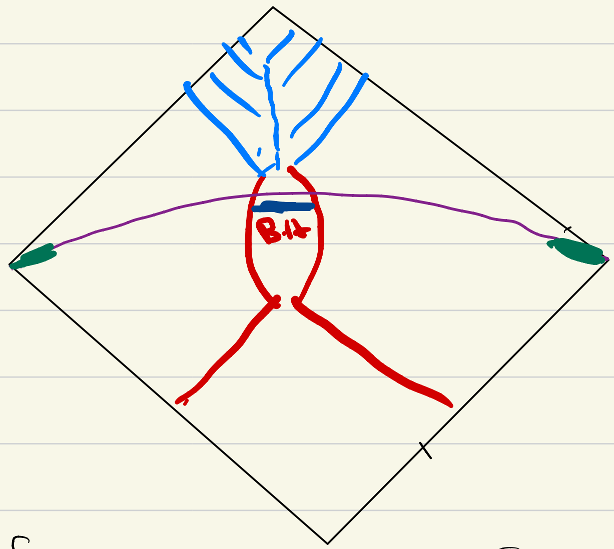
so we shouldn't ask

"how is the information recovered when the black hole evaporates."

the p.HoI suggests that the information can be recovered even **before evaporation**

Note it also suggests that

An error in Hawking's argument was the assumption of the principle of ignorance.



Principle of ignorance suggests the state in the dark blue region can be specified independently of the green region.

Principle of HoI suggests precisely the opposite!

Changing the state inside necessarily changes the state outside.

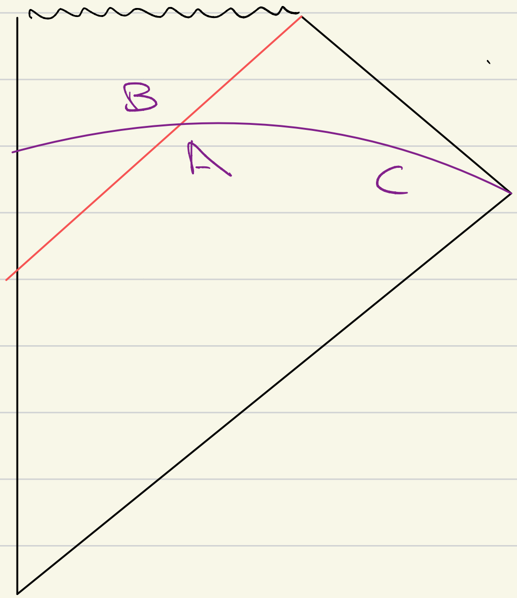
This is because gravity localizes information very differently from LQFTs.

The failure to take this into account is an error in Hawking's argument.

[Explain more]

Relevance for the monogamy paradox

Brief reminder of the monogamy paradox.



If we insist that the Hilbert space on a Cauchy slice factorizes and information in gravity is localized just as in QFTs then we find that

$$|\langle C_{AB} \rangle| > 2$$

and also

$$|\langle C_{AC} \rangle| > 2$$

and so the mongamy inequality

$$\langle C_{AB} \rangle^2 + \langle C_{AC} \rangle^2 \leq 8$$

is violated.

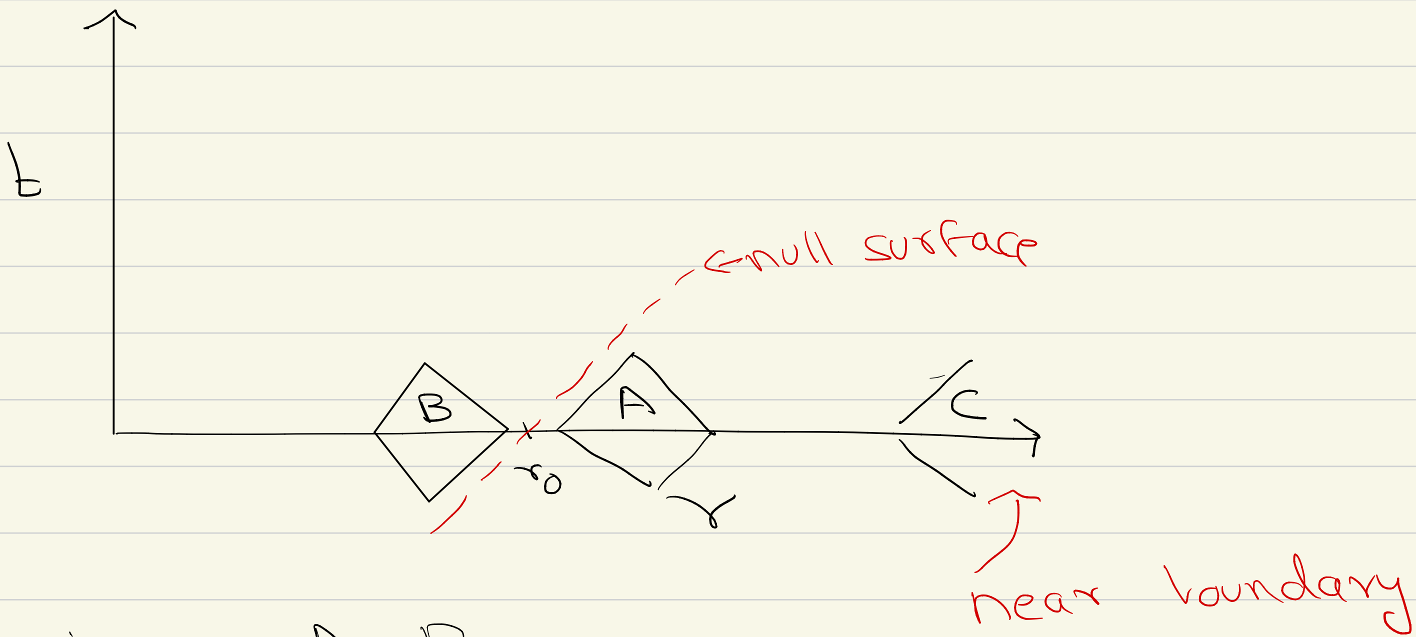
We will now show that if one **insists** that there should be no difference between gravity & local QFTs then one can **construct** a paradox even in empty space!

To avoid IR issues let us return to empty global AdS.

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_{d-1}^2$$

Consider a sphere at some value of $r = r_0$

We define the regions A, B, C as follows



The regions A, B are close together

As we emphasized over several lectures, we can find a set of modes in A, B that are entangled with each other.

As a brief reminder. Let $N = a^+ a$ be the number operator of modes with frequency ω_0 in A and $\tilde{N} = \tilde{a}^+\tilde{a}$ of B

Then we can define

$$A_1 = \sum_{n=0}^{\infty} \left(|2n+1\rangle \langle 2n+1| - |2n\rangle_A \langle 2n| \right)$$

$$A_2 = \sum_{n=0}^{\infty} |2n+1\rangle\langle 2n| + |2n\rangle\langle 2n+1|$$

$$\tilde{A}_1 = \sum_{n=0}^{\infty} |2\tilde{n}+1\rangle \langle 2\tilde{n}+1| - |2\tilde{n}\rangle\langle 2\tilde{n}|$$

$$\tilde{A}_2 = \sum_{n=0}^{\infty} |2\tilde{n}+1\rangle \langle 2\tilde{n}| + |2\tilde{n}\rangle \langle 2\tilde{n}+1|$$

Then set

$$B_1 = \cos\theta\, \tilde{A}_1 + \sin\theta\, \tilde{A}_2 \quad \text{and} \quad B_2 = \cos\theta\, \tilde{A}_1 - \sin\theta\, \tilde{A}_2$$

with $\tan\theta = \dfrac{2e^{-\pi\omega_0}}{1+e^{-2\pi\omega_0}}$

with

$$C_{AB} = A_1 (B_1 + B_2) + A_2 (B_1 - B_2)$$

$$\langle 0 | C_{AB} | 0 \rangle = \frac{2}{1+e^{-\pi\omega_0}} \left(1 + 6 e^{-\pi\omega_0} + e^{-2\pi\omega_0}\right)^{1/2} > 2$$

AdS vacuum

Two important points

a) Here by operators "in" A or B we are using **the same perspective used in the monogamy paradox**. These can be thought of as some gauge-fixed quasi-local operators
[The resolution later will be that these operators are **not reall local.**]

ii) All correlators computed here receive corrections.
Some corrections arise due to the "tuning" function whereas others arise due to to interactions

These **do not** affect the leading result.

We will now find operators in C that will violate the monogamy inequality.

Recall that even in LQFT we can find operators Q_i with the property that

$$Q_1 |0\rangle = |B_1\rangle$$ [Notation $|B_1\rangle = B_1|0\rangle$]

$$Q_2 |0\rangle = |B_2\rangle$$

But we can't just insert these into a CHSH operator because Q_i do **not** have operator norm bounded by 1.

In gravity by combining Q_i and P_0 one can construct bounded observables

$$C_1|0\rangle = |B_1\rangle \quad \text{and} \quad C_2|0\rangle = |B_2\rangle$$

First note that we can construct operators

$$|B_i\rangle\langle 0| = Q_i P_0$$

$$|0\rangle\langle B_i| = P_0 Q_i^\dagger$$

$$|B_i\rangle\langle B_i| = Q_i P_0 Q_i$$

We still need to be careful about getting something with op. norm <= 1

It turns out the right combination is

$$C_i = \frac{\left(|B_i\rangle\langle 0| + |0\rangle\langle B_i| - \langle B_i\rangle |0\rangle\langle 0| - \langle B_i\rangle |B_i\rangle\langle B_i| \right)}{\langle B_i^2\rangle - \langle B_i\rangle^2}$$

↑
EXPLAIN IDEA

This has the property

$$C_i |0\rangle = |B_i\rangle$$

and also

$$\|C_i\| = \langle B_i^2\rangle \leq 1.$$

Now it is clear that we can construct

$$C_{AC} = A_1(C_1 + C_2) + A_2(C_1 - C_2)$$

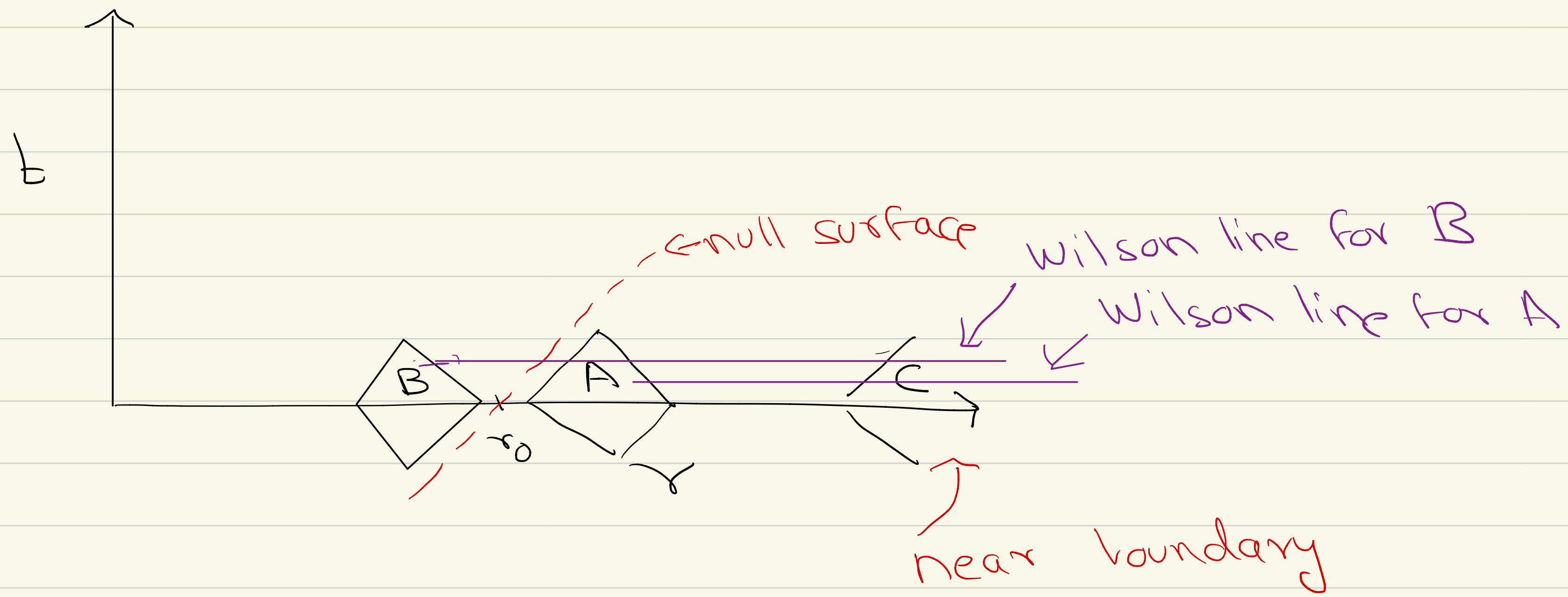
$$\langle C_{AC} \rangle = \langle 0 | A_1 (C_1 + C_2) | 0 \rangle$$
$$+ \langle 0 | A_2 (C_1 - C_2) | 0 \rangle$$
$$= \langle C_{AB} \rangle$$

[Using action of C_i on $|0\rangle$]

So we have

$$\langle C_{AB} \rangle^2 + \langle C_{AC} \rangle^2 = 2 \langle C_{AB} \rangle^2 > 8!$$

The resolution to this paradox is clear



We pretended the operators A, B were "localized" in regions A, B.

But, in reality they must be dressed to the boundary.