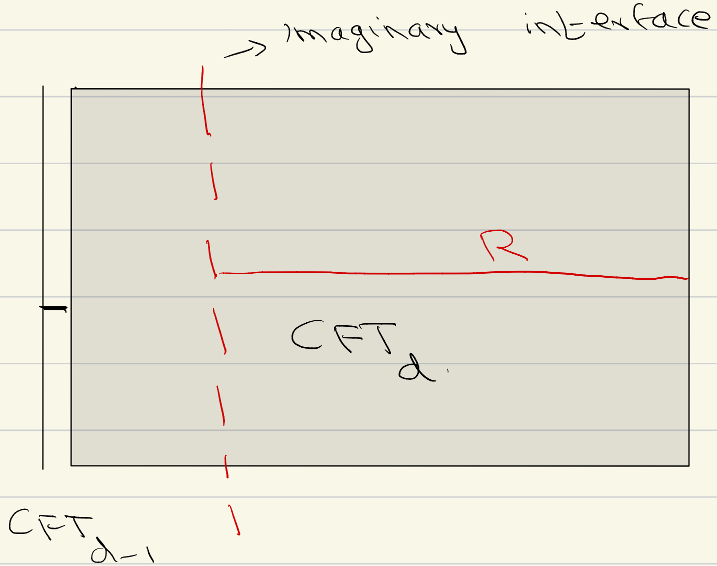
24 March 2021

Lecture 19: Islands on Branes

Last time we discussed how in theories where AdS was coupled to a **non-gravitational** bath, islands could qualitatively lead to the emergence of a Page curve.

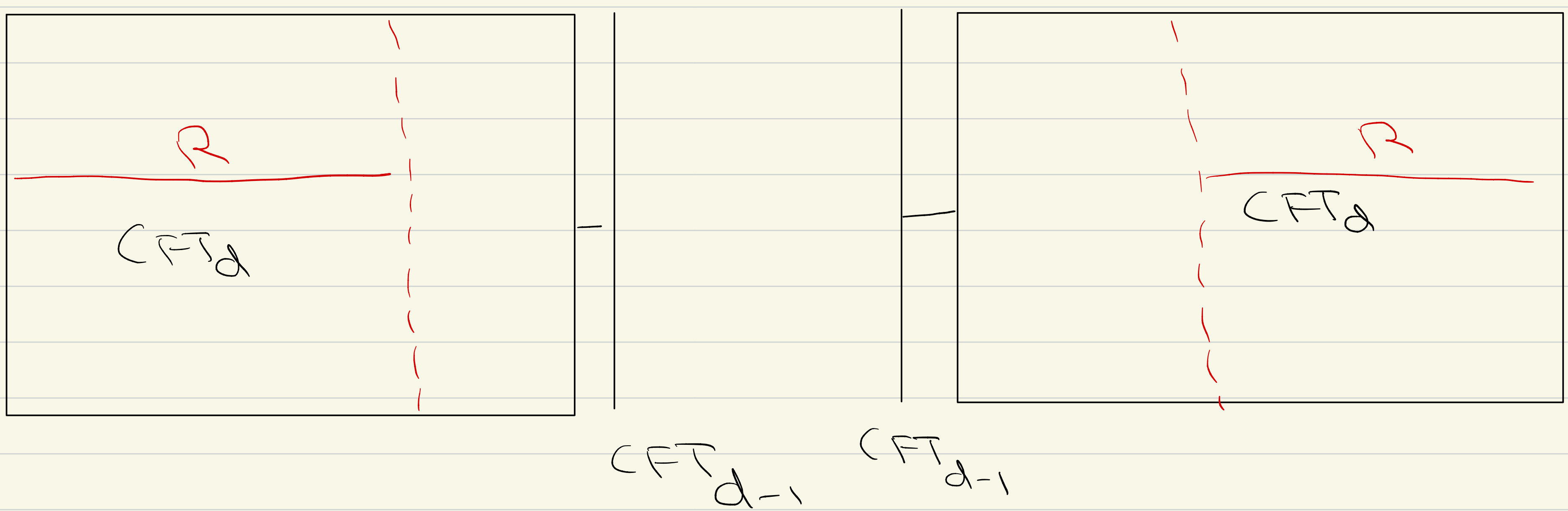


This Page curve has a precise nongravitational interpretation.
Some of the concomitant language like "collecting information in the Hawking radiation" is **imprecise**

Also these calculations are hard to make precise and one must wave one's hands a little.

A more precise setup is as follows.

Lets take **two copies** of this system



Initially we entangle the two CFTs with each other.

Note the left and right systems are entangled but noninteracting

But each CFT_{d-1} is interacting with its own bath.

We can prepare the system so that it is in a thermofield double state at $t=0$

$$\sum e^{-\beta E/2} |E\rangle_L \otimes |E\rangle_R$$

where $|E\rangle_L$ and $|E\rangle_R$ are energy eigenstates of the left and right [CFT_{d-1} + CFT_d bath] system.

In such systems, the density matrix on each side is thermal.

The whole state is also invariant under

$$e^{i(H_L - H_R)T}$$

But it is **not invariant** under

$$e^{i(H_L + H_R)T}$$

We can still ask non-trivial questions about correlations between left and right.

In these setups, a "paradox" is constructed as follows

Consider entanglement entropy of union of part of left bath and right bath.

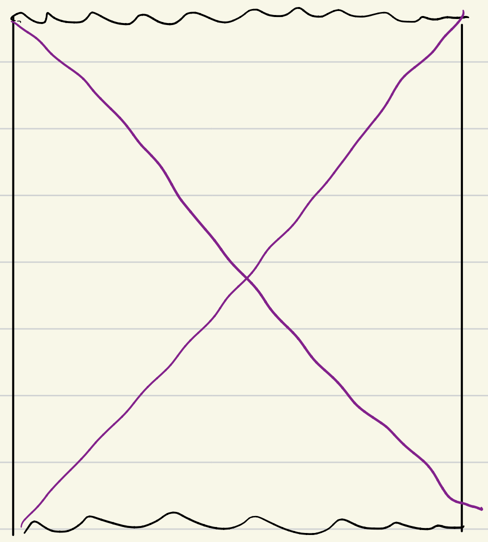
If we do a naive/wrong computation of this entropy using the bulk dual, it grows without bound.

But this is unphysical and so this growth must be cut off.

[Note this is a paradox involving a nongravitational system, and so different from other versions of info. paradox.]

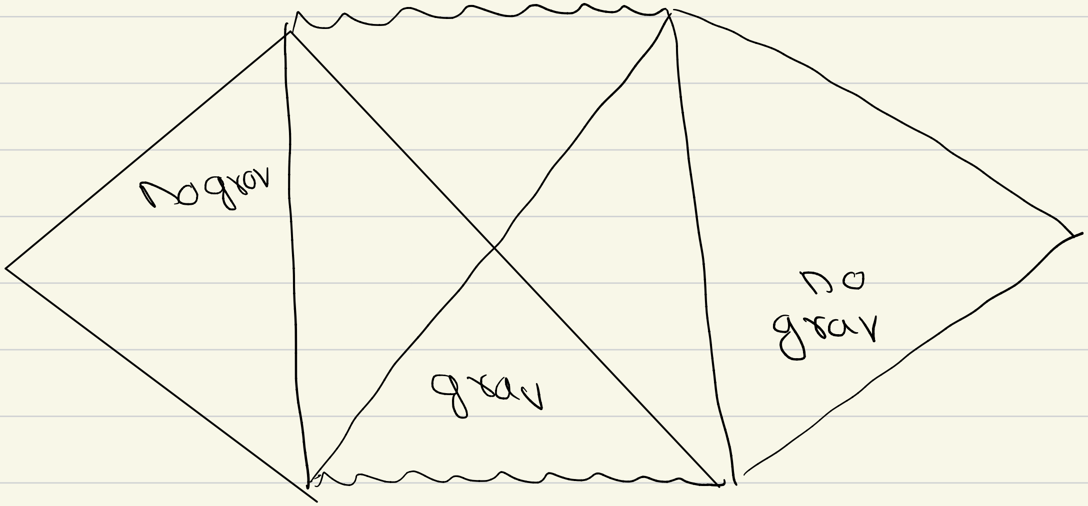
In the bulk, we need to introduce "islands" $\longrightarrow$ right answer in the nongravitational dual.

The bulk dual is as follows.
Consider an **eternal black hole** in AdS



Lets now couple it to an external

system at the **same temperature**

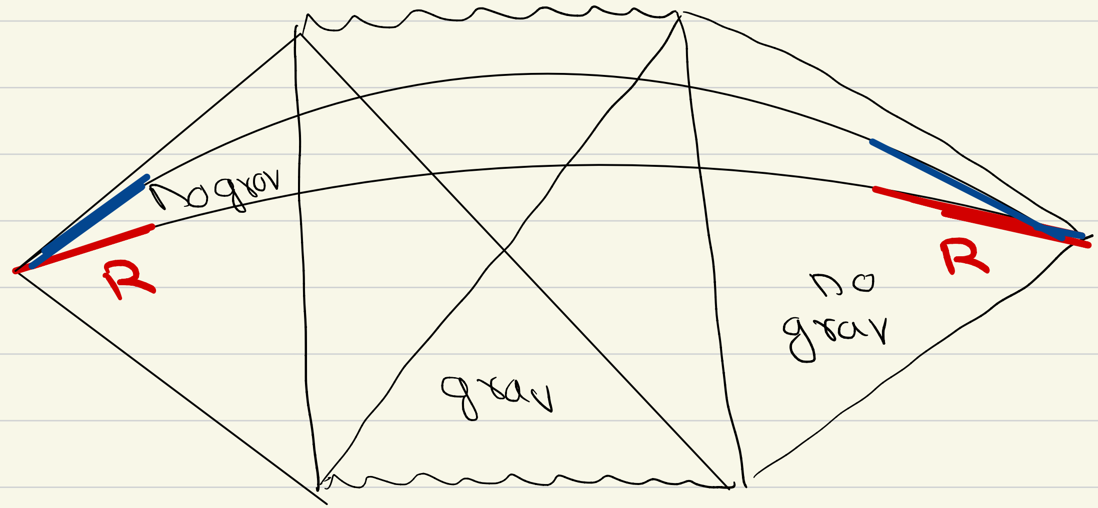


The eternal black hole geometry is "static" if we increase t_L and decrease t_R.

To get interesting time evolution we need to increase both.

But we can set up a puzzle as follows

Consider a Cauchy slice and let R be the union of segments on the two sides.



Now push time "up" on both sides (so R evolves from red to blue).

As we do this, the length of the Cauchy slice in the interior stretches <u>without</u> <u>bound</u>

If one uses naive techniques to compute the entropy of R, one would find

This $S_{semi-cl}(R)$ involves an **incorrect calculation**!

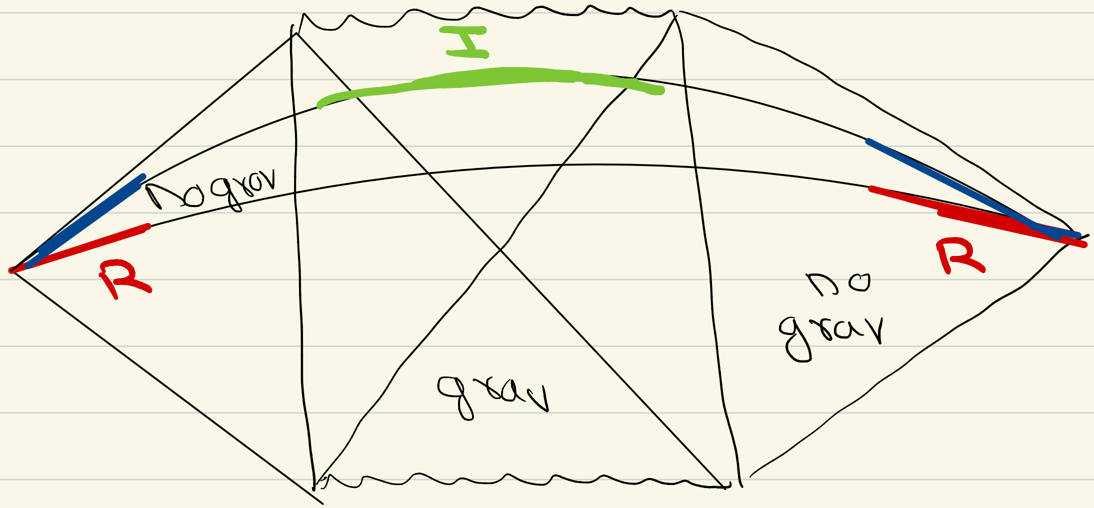
We will later see the precise "mistake" that needs to be made to obtain this answer

But we know that the full eternal black hole is described by a CFT_{d-1}, which has **a finite no. of dof**.

so $S_{semi-cl}(R)$ cannot increase without bound.

This is a contradiction between the finite density of states in the dual and the fact that the b.h. sometimes seems to require an unbounded density of states! This will appear later as well.

This puzzle is resolved by the appearance of an island at late times.

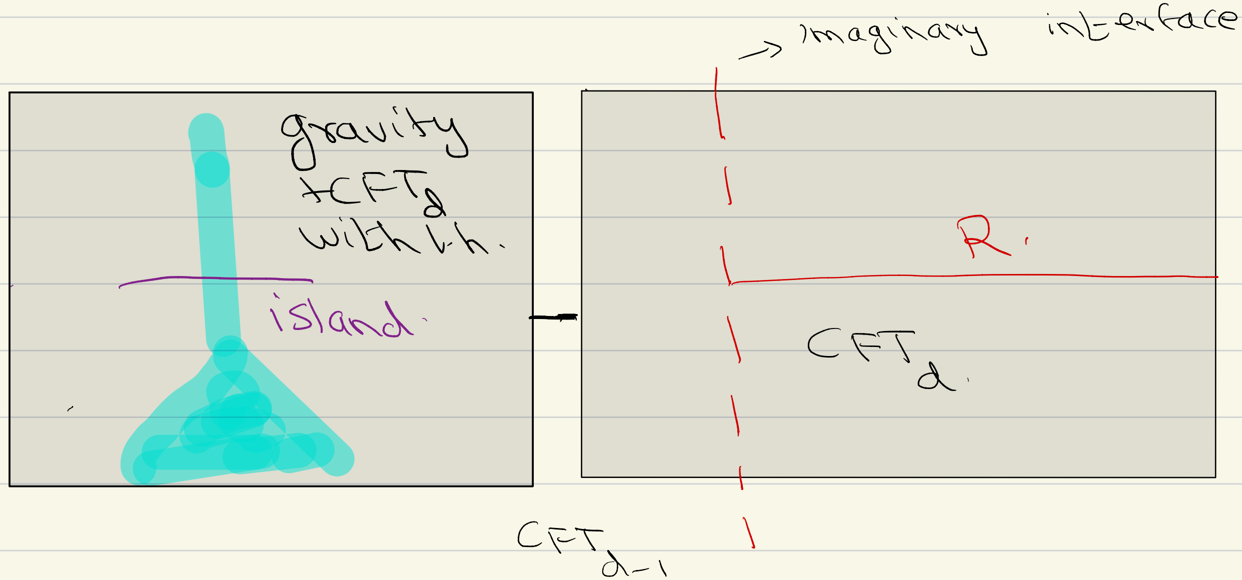


The appearance of I purifies R and so cuts off the growth of S(R).

This can be made precise in 1+1 D or using braneworld models in higher D.

The idea is as follows.

So far we have considered gravity coupled to a CFT_d.



Now take the CFT_d itself to be holographic

This leads to a scenario first studied by Karch and Randall

We find a system with **three** descriptions

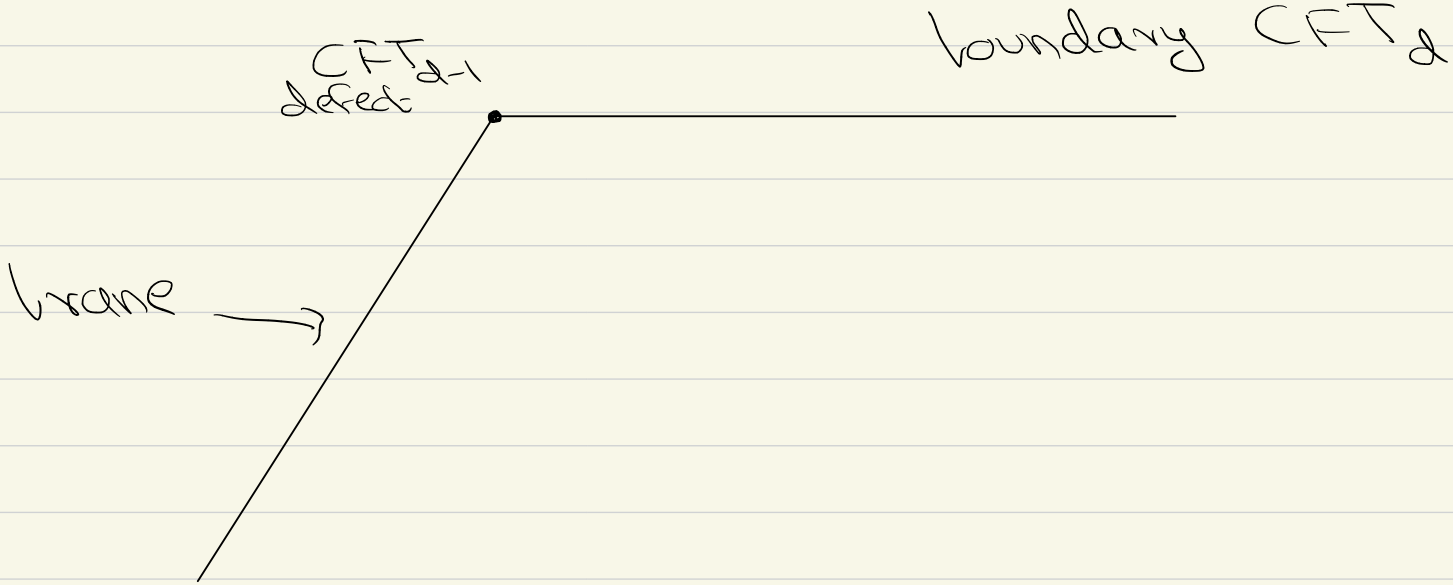
Recall we already have 2 descriptions

1) A CFT_d that ends on a defect with a CFT_{d-1}

2) A CFT_d coupled to gravity in AdS and then further coupled to a nongravitational bath.

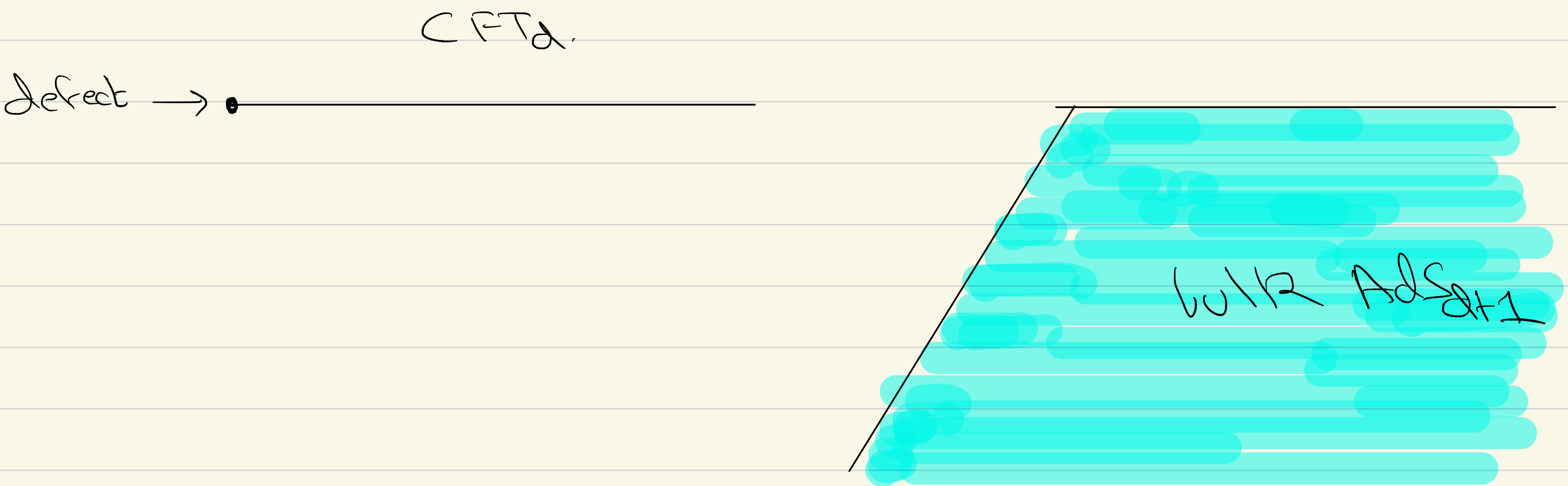
We now find a third description, which is

3) A theory of gravity in AdS_{d+1} where the WR AdS is terminated by a brane.



In fact, although we started from description 2, we will now mostly use it only for words.

We can in principle think directly of the duality between 1 and 3



On the boundary, we impose some boundary conditions
In the bulk we do the same.

in the bulk we just write down the **leading term in a derivative expansion** for the brane action. This is just the brane tension.

$$-\int_{\text{brane}} \sqrt{h}\, T$$

where h_{ab} is the induced metric on the brane.

For the bulk geometry to be a classical solution, we need a version of the junction conditions to be satisfied at the brane

$$K_{ab} - K h_{ab} = 8\pi G T$$

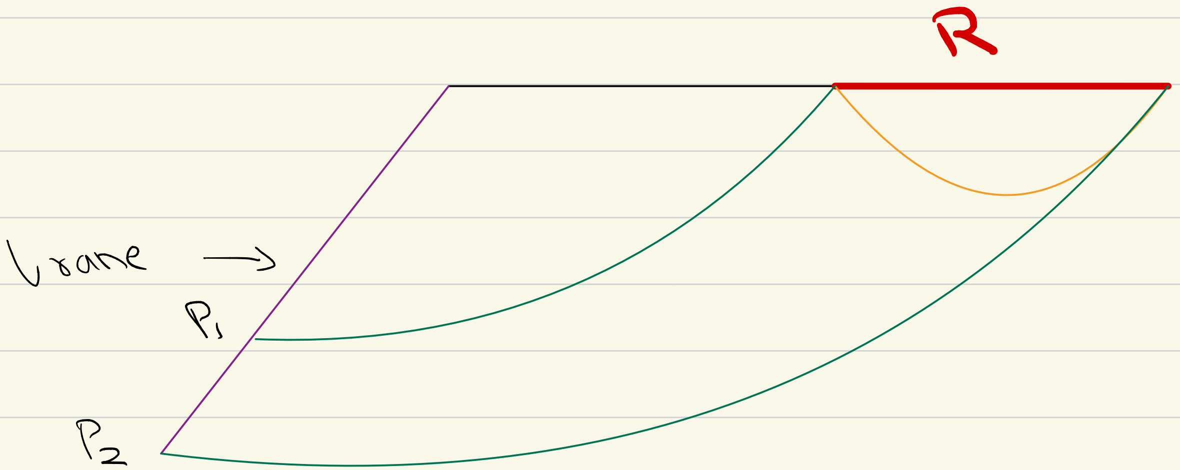
where K_{ab} is the extrinsic curvature of the brane.

It is the need to satisfy this equation that tells us that we cannot arbitrarily insert branes in whatever geometry we wish.

But we will consider a few simple solutions, where solutions with bulk branes can be found easily.

Now the prescription for computing E.E. becomes very straightforward.

Say we want to compute the entropy of a region R.



Then

1) We are allowed to compute the area of ordinary minimal surfaces, such as the orange one.

2) In addition we are also allowed to consider the green surfaces that end

on the brane. **These are islands!**

3) For surfaces that end on the brane, we are instructed to minimize over the end-points P_1 and P_2.

4) In imposing the homology constraint, the brane is not counted as a "boundary"
[Homology constraint: region between R and the RT surface should have no other boundaries.]

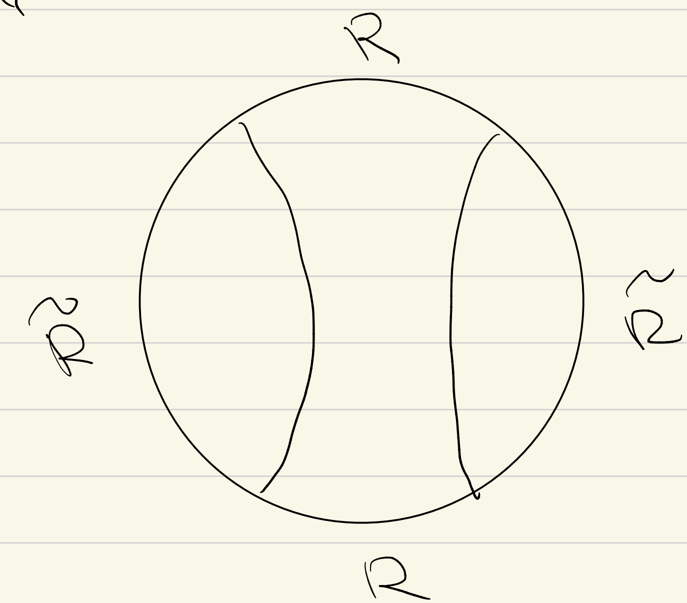
The justification is that the brane is in a region of dynamical gravity

5) There is no "deep derivation" of these rules. They are plausible and they work!

This also perhaps provides the cleanest understanding of islands

Islands correspond to a nontrivial entanglement wedge.

Recall that this also happens in empty space



We will now do one example in detail.

Consider the following AdS black hole geometry

$$ds^2 = \frac{1}{z^2}\left(-h(z)\,dt^2 + \frac{dz^2}{h(z)} + dy^2 + dx_i^2\right)$$

$i = 1 \dots d-2$

and

$$h(z) = 1 - \frac{z^d}{z_0^d}$$

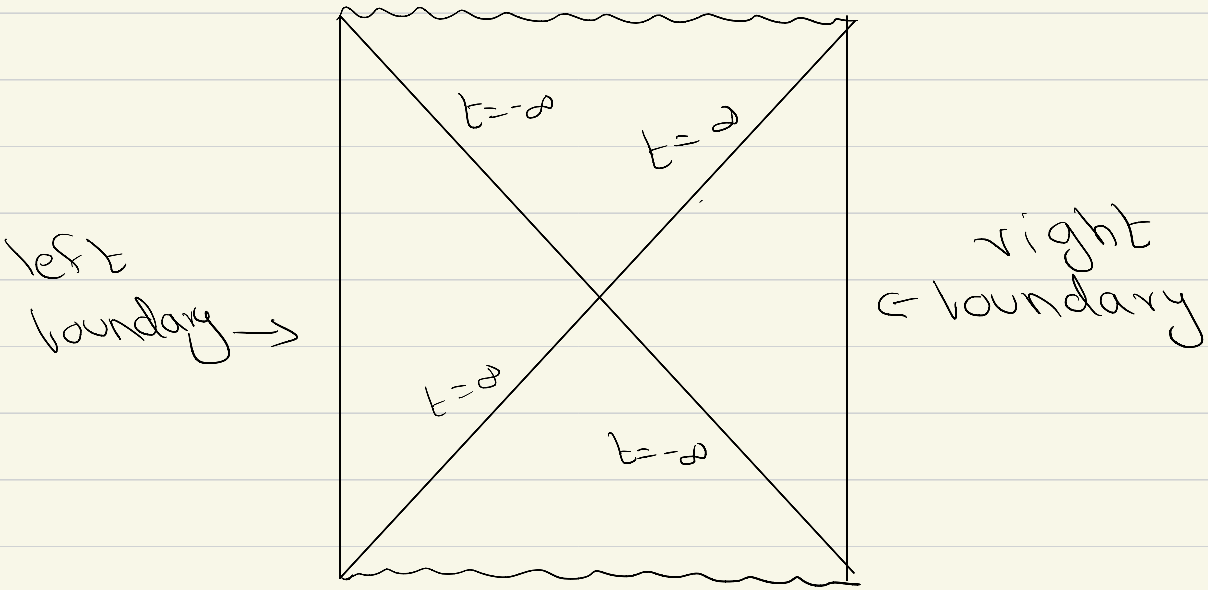
In this geometry, we can put a brane at $y = 0$.

Some comments on the geometry.

This is an eternal black hole.

So it has **two** asymptotic regions.

If we fix y and x_i we get the following Penrose diagram



there is clearly an isometry under Schwarzschild

$$t \to t + \text{const}$$

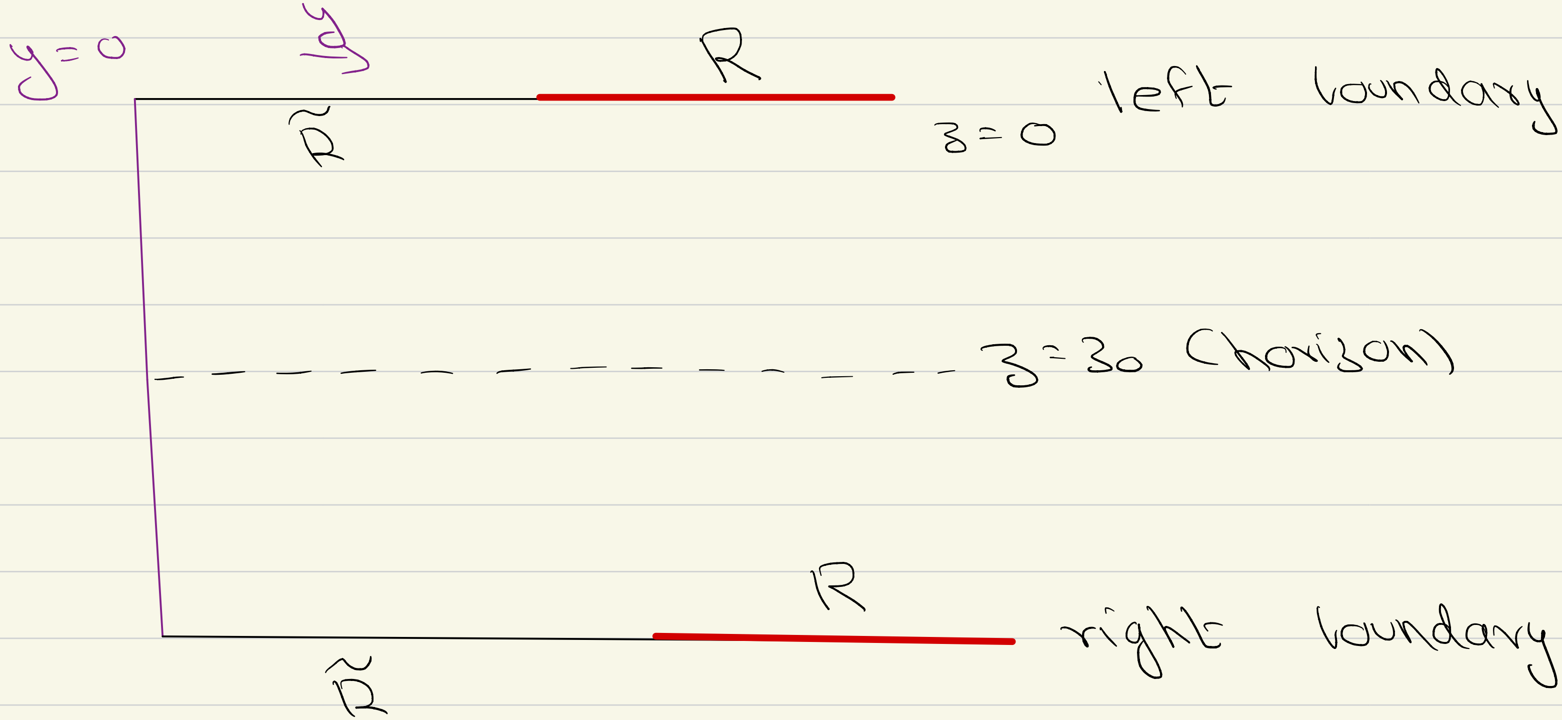
But note that physical time runs in opposite directions in the two asymptotic regions

So the isometry is really under

$$e^{i(H_L - H_R)\tau}$$

If we push time "forward" on both sides, this action is nontrivial.

Now consider a $t =$ const slice.



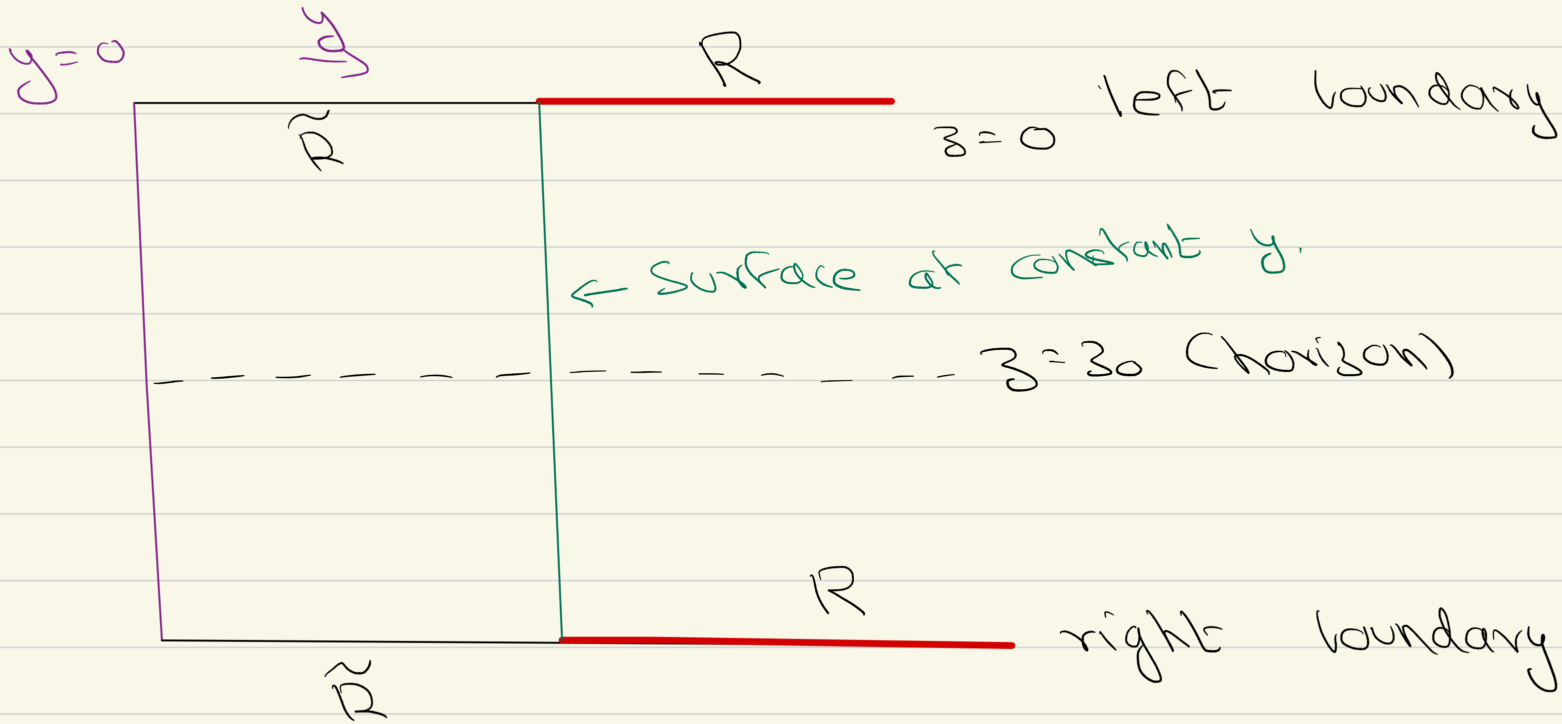
We are interested in computing the entanglement of

segment of left $\cup$ segment of right

with its complement and seeing how this varies as **both segments are pushed forward in time.**

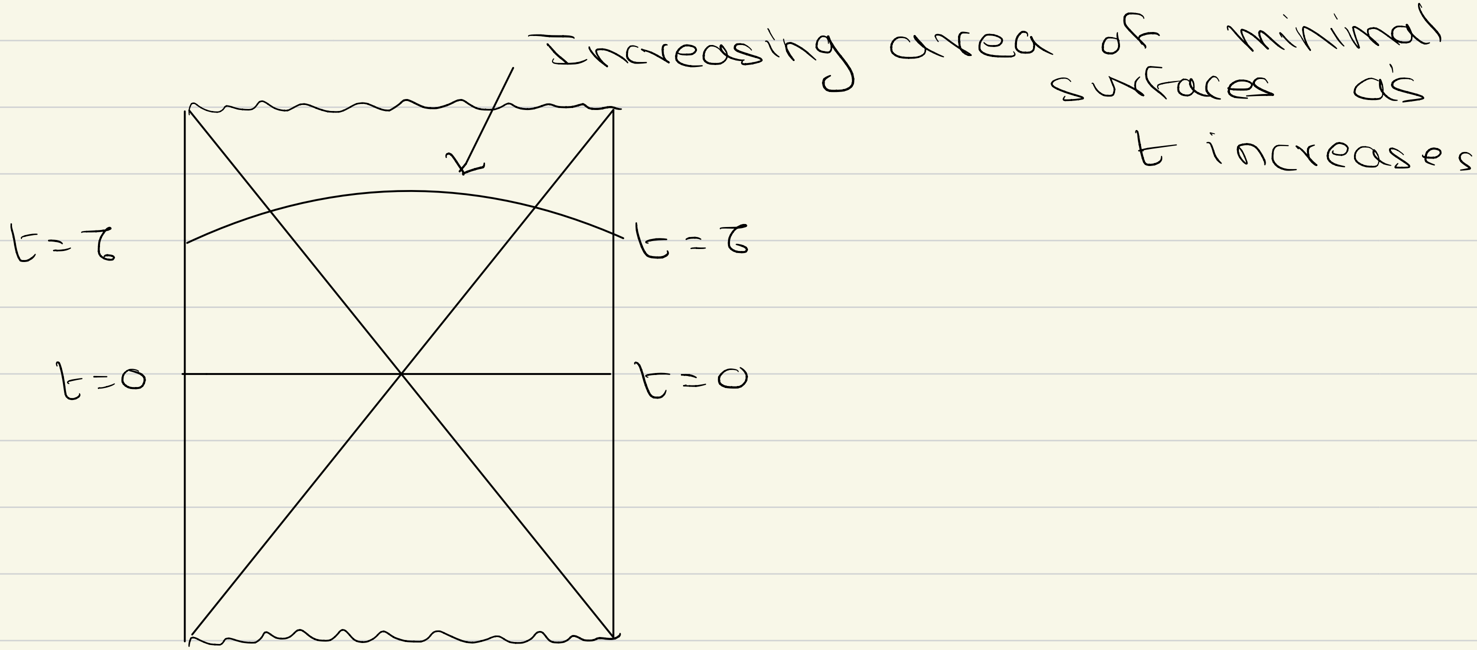
Let us compute this holographically.

First we will consider a surface that travels at constant y from one boundary to the other.



As we push R up in time on both directions, this surface may have to move in t also.

We expect that the area of the surface will grow in time based on intuition from the bulk.



The action to be minimized is:

$$A = \int \frac{1}{z^{d-1}} \sqrt{-h(z) + \frac{\dot{z}^2}{h(z)}} \, dt$$

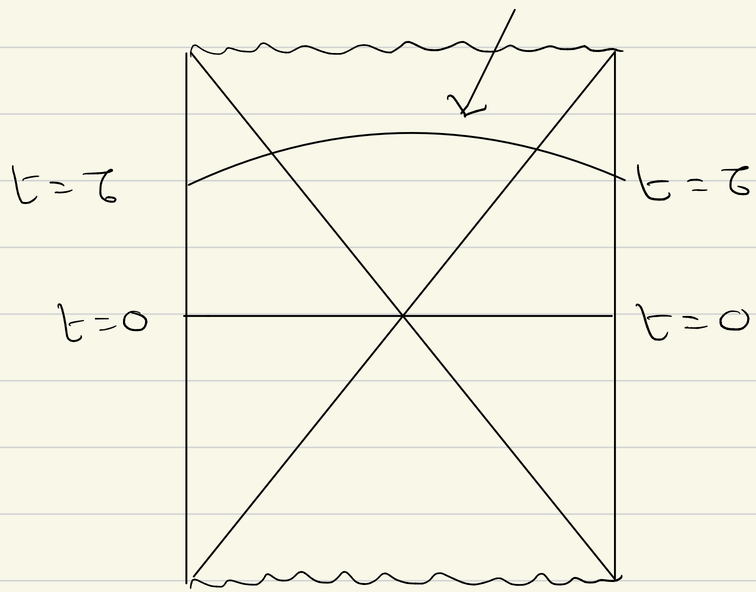
Note this is not a length and this extra factor of z^{d-1} is important.

Since we have a symmetry in that the "action" is invariant under translations of t, we find a conserved quantity:

$$C = \dot{z} \frac{\partial L}{\partial \dot{z}} - L = \frac{h(z) z^{1-d}}{\sqrt{\frac{\dot{z}^2}{h(z)} - h(z)}}$$

From here we see that as one approaches the horizon, $h(z)=0$, we have $\dot{z}=0$.

Outside the horizon, z increases and t also increases. Inside the horizon z decreases and t increases



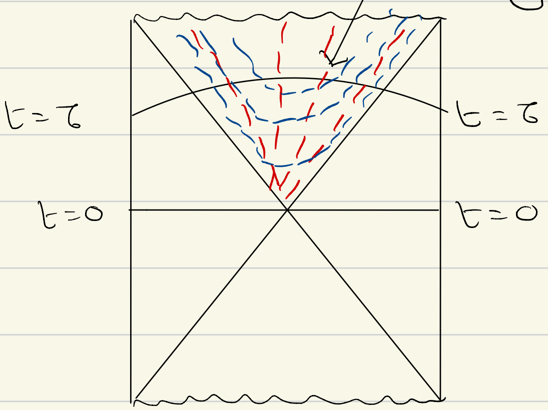
So we set

$$\dot{z} = \frac{h(\vec{z})}{c}\sqrt{c^2 + z^{2(1-d)}h(z)}, \quad z < z_h$$

$$\dot{z} = -\frac{h(\vec{z})}{c}\sqrt{c^2 + z^{2(1-d)}h(z)}, \quad z > z_h$$

We do not need to integrate all the way to the "other side"

$\dot{z}$ goes to 0 here



[Recall const t surfaces are the str t lines and const z are the hyperboloids]

We can integrate up to the point where $\dot{z} = 0$ and write

Call this point z_s which solves

$$c^2 = -h(z_s)\, z_s^{2(1-d)}$$

[Note $h(z_s)$ is $-$ve inside horizon]

$$A = 2 \lim_{\varepsilon \to 0} \left[\frac{-1}{(d-2)\varepsilon^{d-2}} + \int_{\varepsilon}^{z_s} \frac{dz}{|\dot{z}|} \frac{1}{z^{d-1}} \sqrt{-h(z) + \frac{\dot{z}^2}{h(z)}} \right]$$

and

$$t_{diff} = \lim_{\delta \to 0} \left[\int_0^{z_0 - \delta} \frac{dz}{\dot{z}} + \int_{z_0 + \delta}^{z_s} \frac{dz}{\dot{z}} \right]$$

The regulator Σ is required to cut off the UV-divergence in the area near the boundary.

The regulator δ is required since $t \to \infty$ near the horizon and then comes down again.

Now note that $\dot{z}$ goes to 0 near the horizon and also in the interior.

Near the horizon we do not get any divergence either in the area (since $h(u)$ also goes to 0)

Nor in t_{diff} [Due to the principal value prescription imposed by δ.]