Black Hole Information Paradox
Lecture 1 - 13 January 2021.

Logistics

1) protocol for questions/comment.
2) Tutorials from next week
3) Lectures on Youtube immediately afterwards.
   → not livestreamed.
4) Assignments and notes on class website.

We will closely follow arXiv:2012.05770 that was partly written for this course!

Overview of course

~ 26 lectures

Hawking's original paradox $\longrightarrow$ Thermalization and exp-small corrections

Paradoxes about the interior of evaporating black holes $\longrightarrow$ Holography of info Islands

paradoxes with large black holes $\longrightarrow$ Mirror operators state-dependence Firewalls

But this is a course, not a seminar!

So we will often go slowly and make technical detours, including in this lecture.

The physical phenomenon that underlies the information paradox is HAWKING RADIATION.

Black holes emit radiation with a temperature proportional to the surface gravity

We will start by understanding this physics.

The physical points to emphasize are:

a) Hawking rad is a robust prediction. Relies on short-distance correlators of fields and global late-time properties of black holes

→ key issue in many paradoxes

b) Same derivation implies existence of entangled modes across the horizon.

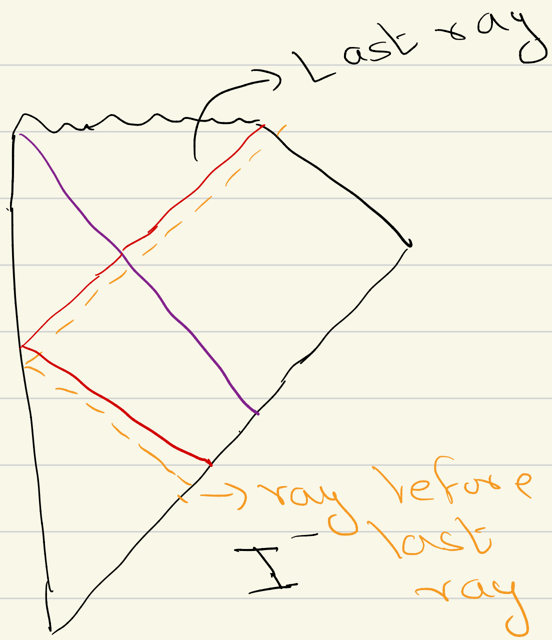
Two commonly provided derivations are

1) Hawking's original derivation

2) Rindler $\leftrightarrow$ Minkowski Bogoliubov transformation

Sketch:

<u>Hawking</u>



Hawking's idea was to use geometric optics to determine how rays from I^- propagate to I^+.

Hawking found that one could compute the final state on I^+ if one put the modes on I^- in the vacuum.

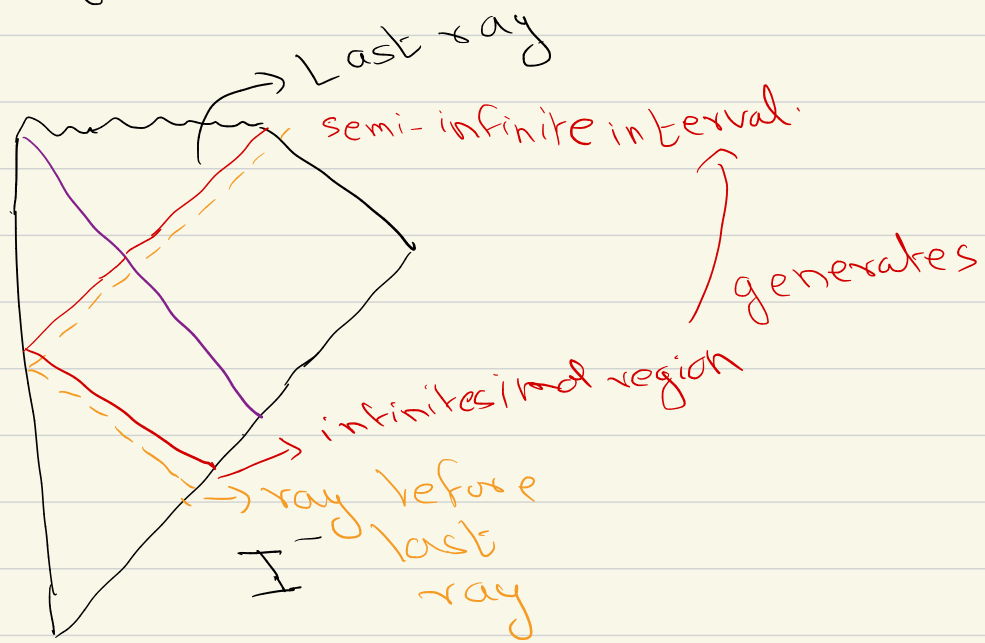
Advantages

clever

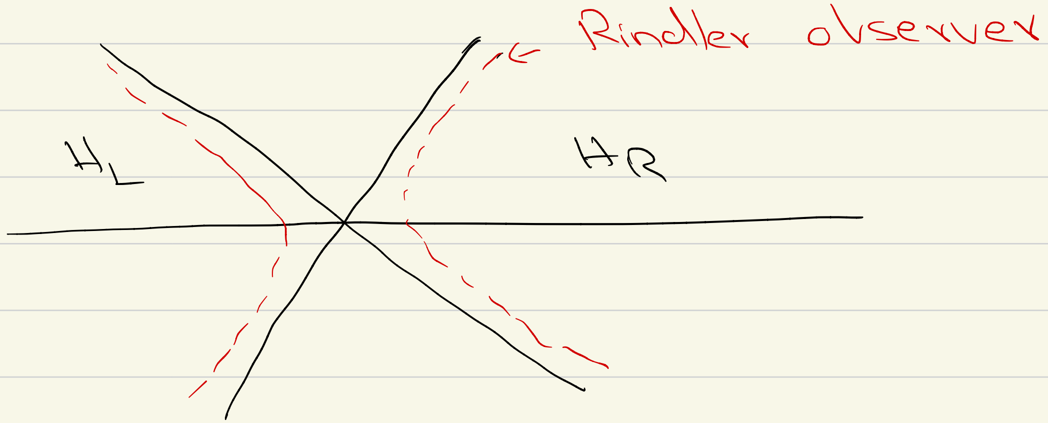
gives physical insight

- not clear how to correct this systematically

trans Planckian problem



Another common argument is to provide the analogy with Rindler space



The Minkowski vacuum looks thermal for a Rindler observer.

Advantage	Question
Technically simple	How much do the global aspects of geometry matter compared to local aspects?

We will start by rederiving Hawking radiation

This leads us to our first detour

"Quantum Fields near a null surface"

We will later apply this to black holes.

The result we want to show is:

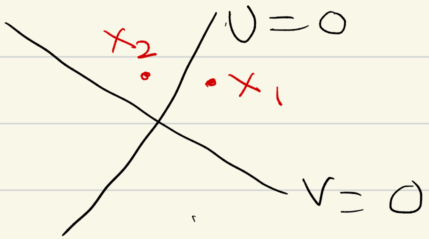
"Across any null surface, one can isolate "local" QFT degrees of freedom with universal entanglement."

Later we will show that

in a B.H. spacetime, these local modes near the horizon get related to global modes.

Consider the vicinity of a smooth point is some spacetime. We can always introduce coords so that

$$ds^2 = -dU\,dV + \delta_{\alpha\beta}\, dy^\alpha dy^\beta$$



y^α are transverse coords

This is true in arbitrary dimensions.

Now lets consider correlators of quantum fields close to $U=0$.

In any nonsingular state, we have

$$\langle \phi(x_1)\,\phi(x_2)\rangle = \frac{N}{|x_1 - x_2|^{d-1}}$$

$$\langle \phi(x_1)\,\phi(x_2)\rangle = \frac{N}{|x_1-x_2|^{d-1}} = \frac{N}{\left(-\delta u\,\delta v + \delta_{\alpha\beta}\,\delta y^\alpha \delta y^\beta\right)^{(d-1)/2}}$$

Some physics points.

1) We are assuming that

$$|x_1 - x_2| \ll \lambda_{\text{curvature}}$$

2) Also

$$|x_1 - x_2| \ll \frac{1}{m}$$ if m is the mass of the field

3) But

$$|x_1 - x_2| \gg \lambda_{Pl}.$$

or the UV scale at which the EFT breaks down.

The constant N can be fixed using a canonically normalized free field in flat space i.e. with Lagrangian

$$L = \frac{1}{2}(\partial_\mu \phi)^2$$

$$N = \frac{\Gamma(d-1)}{2^d \pi^{d/2} \Gamma(\frac{d}{2})}$$

in $(d+1)$ - spacetime dimensions

Even though we compute N for a special case, it holds for other spacetimes in the limit of interest.

There are, of course, subleading terms in the correlator but they will not be important.

Now, we manipulate this correlator. The significance of these manipulations might initially be unclear but will soon become apparent.

Differentiate w.r.t. v_1 and also v_2.

We get [in this limit]

$$\langle \partial_{v_1} \Phi(x_1)\, \partial_{v_2} \Phi(x_2) \rangle = -\frac{d^2-1}{4} N \frac{(\delta v)^2}{|x_1 - x_2|^{d+3}}$$

Explain factors.

Now we take $\delta v \to 0$ although we do not have to take it to be exactly zero but merely smaller than all other separations.

What we have here is

$$\lim_{\delta v \to 0} \frac{(\delta v)^2}{(-\delta u \delta v + \delta y^\alpha \delta y^\alpha)^{(d+3)/2}}$$

It might seem that this goes to 0

But notice that the denominator contributes exactly when $|\delta y| = 0$.

In fact the contribution is precisely a delta function in the transverse coordinates.

This can be seen as follows. consider.

$$\int \frac{(\delta v)^2 \; d^{d-1}\delta y^\alpha}{(-\delta u \delta v + \delta y^\alpha \delta y^\alpha)^{(d+3)/2}}$$

Lets take the limit $\delta v \to 0$ so that $\delta u \delta v < 0$ [this way we don't have to be careful about an $i\varepsilon$ prescription]

Then change variables to $\dfrac{\delta y^\alpha}{\sqrt{-\delta u \delta v}} = \delta \tilde{y}^\alpha$

The integral becomes

$$\frac{1}{(-\delta u \delta v)^{(d+3)/2}} \times (-\delta u \delta v)^{(d-1)/2} \times (\delta v)^2 \times \int \frac{d\,\delta\tilde{y}^\alpha}{[1 + \delta\tilde{y}^\alpha \delta\tilde{y}^\alpha]^{(d-3)/2}}$$

$$= \frac{1}{(\delta u)^2} \int \frac{d\,\delta\tilde{y}^\alpha}{[1 + \delta\tilde{y}^\alpha \delta\tilde{y}^\alpha]^{(d-3)/2}}.$$

Since the integral is finite and since clearly the integrand vanishes when $\delta y^\alpha \neq 0$ we conclude

$$\lim_{\delta v \to 0} \langle \partial_{u_1} \phi(x_1)\, \partial_{u_2} \phi(x_2) \rangle = -\frac{1}{4\pi} \frac{1}{(u_1 - u_2 - i\varepsilon)^2} \delta^{d-1}(\delta y).$$

Fixing const and Fixing $i\varepsilon$ are assignment questions

This is a key formula.

The next step is to integrate $\partial_u \phi$ against an appropriate spacetime function to extract modes

Morally we would like to extract modes by doing the integral

$$\int \partial_u \phi \, (-u)^{-i\omega} \quad \text{[approx expression]}$$

note that the
surfaces of constant phase are
surfaces of constant V

so a and $\tilde{a}$ represent "right movers"
on opposite sides of the horizon