25 March 2021

Lecture 20: Gravity in the Bath

Yesterday, there was a question about the contribution to the entropy from the area on the brane.

This is discussed in detail in arxiv:2006.04851 (see Appendix A)

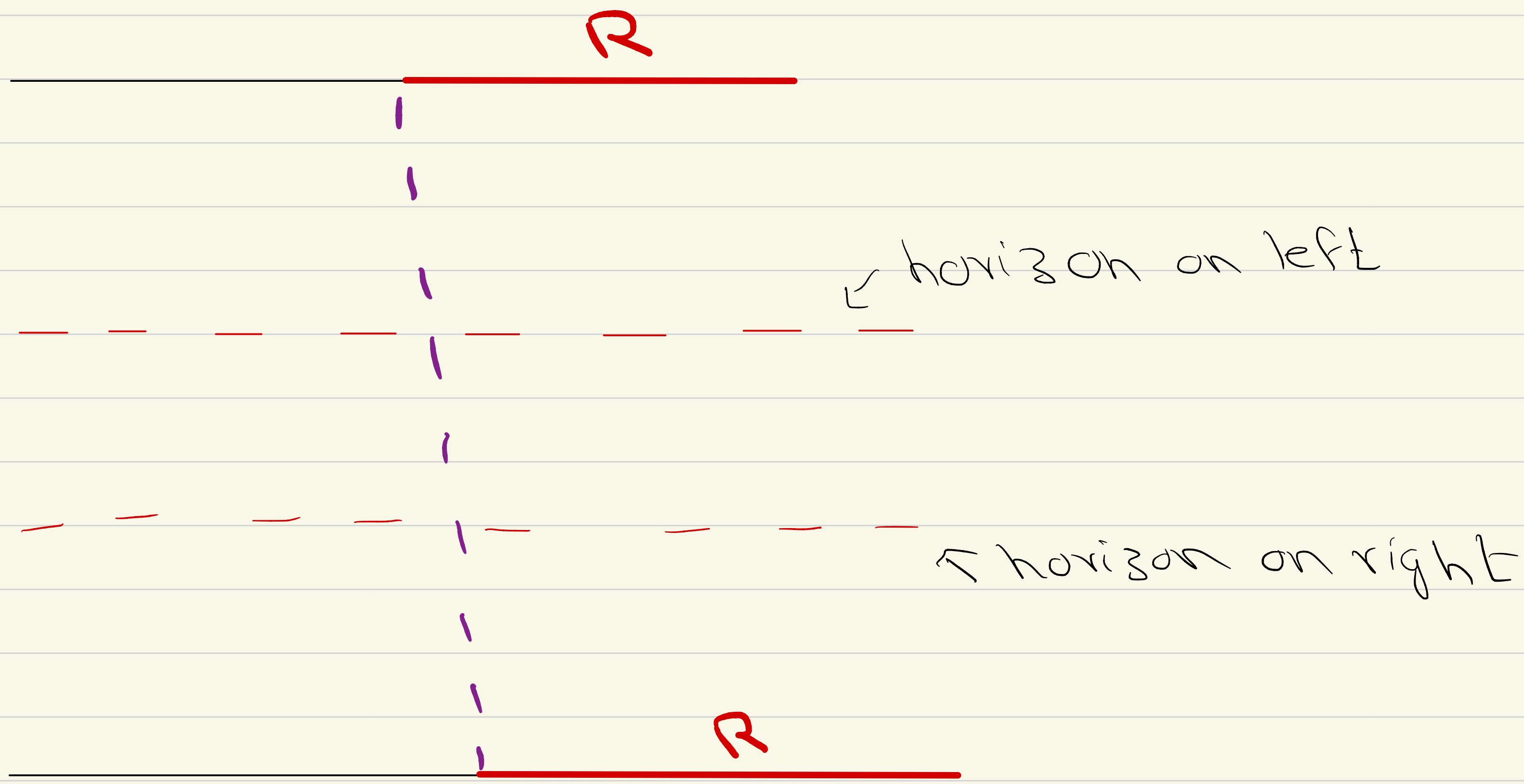
$$S_{EE}(R) = \min\left[\operatorname{ext}\left(\frac{A(\gamma)}{4G_{bulk}} + \frac{A(\gamma \cap \text{brane})}{4G_{brane}} \right)\right]$$

where the second term arises if we add a so-called DGP term on the brane.

$$S_{DGP} = \frac{1}{16\pi G_{brane}} \int_{brane} \sqrt{-h}\, \tilde{R}(h)$$

We are not considering such a term for simplicity.

Yesterday, we computed the parameters of one surface that contributes to the entropy



We found that

$$\dot{z} = \frac{h(z)}{c}\sqrt{c^2 + z^{2(1-d)} h(z)}\ , \quad z < z_h$$

$$\dot{z} = -\frac{h(z)}{c}\sqrt{c^2 + z^{2(1-d)} h(z)}, \quad z > z_h$$

$$A = 2 \lim_{\varepsilon \to 0}\left[\frac{-1}{(d-2)\varepsilon^{d-2}} + \int_{\varepsilon}^{z_s} \frac{dz}{|\dot{z}|}\frac{1}{z^{d-1}}\sqrt{-h(z) + \frac{\dot{z}^2}{h(z)}}\right.$$

and

$$\left. t_{diff} = \lim_{\delta \to 0} \int_0^{z_0 - \delta} \frac{dz}{\dot{z}} + \int_{z_0+\delta}^{z_s} \frac{dz}{\dot{z}} \right]$$

$$c^2 = -h(z_s)\, z_s^{2(1-d)}$$

When $\dot{z} \to 0$ in the interior we see that for generic C

$$\dot{z} \sim \sqrt{z - z_s} \quad , \text{ near } z = z_s$$

where z_s is the turning point.

Recall,

$$\dot{z} = -\frac{h(z)}{C} \sqrt{C^2 + z^{2(1-d)} h(z)}, \quad z > z_h$$

On the other hand.

$$-z^{2(1-d)}\, h(z)$$

also has a maximum value.

$$-z^{2(1-d)} h(z) = \left(\frac{z^d}{z_0^d} - 1\right) \frac{1}{z^{2(d-1)}} = \frac{1}{z_0^d\, z^{d-2}} - \frac{1}{z^{2d-2}}$$

So the max is at

$$\frac{1}{z^{d-1}} \left[\frac{-(d-2)}{z_0^d} + \frac{(2d-2)}{z^d} \right] = 0$$

If C is such that the turning point is near the maximum then

$$z \sim z - z_s ;$$

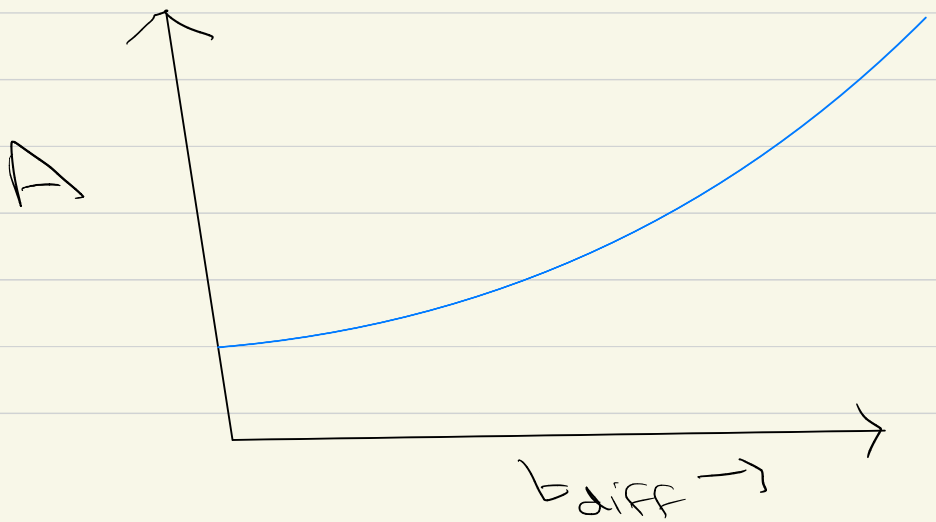
So as we increase c to the max allowed value of

$$c^2_{max} = \left\{ -z^{2(1-d)} h(z) \right\}_{max}$$

we see that both A and t_{diff} increase in an unbounded manner.

Asymptotically we have

$$A \propto t_{diff}$$



This can now be taken to be the "wrong" calculation of $S_{semi-cl}(R)$.

Even after regulation, it seems to increase in an unbounded manner.

This is a puzzle that will be resolved by the appearance of a new RT surface.

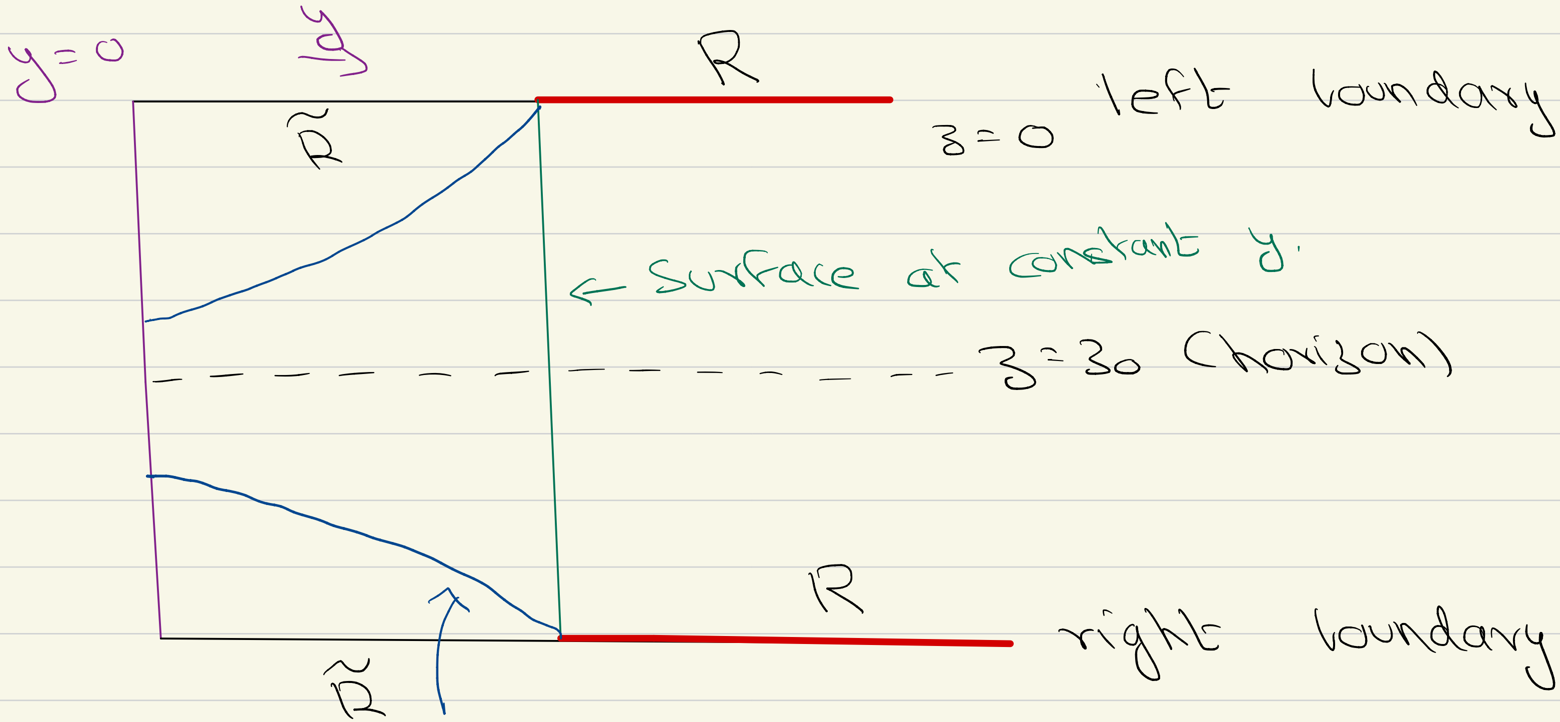
This puzzle has to do with the geometry of the interior, so it can be thought of as a version of the information paradox in this setting.

Another RT surface.

An analysis of this kind was done earlier in arXiv:1303.1080 [before "islands"]

The idea is that this growing RT surface is eventually replaced by another surface.

This is the island surface.



this surface might have lower area at late times.

To find this surface we look for a curve,

$$y(z)$$

that runs from the boundary to the brane.

The area we need to extremize now is

$$A = \int dz \, \frac{1}{z^{d-1}} \sqrt{\frac{1}{h(z)} + y'(z)^2}$$

Remember that we also need to ensure the surface is extremal when we vary the end-point on the brane.

If the surface intersects $y=0$ at $z=z_s$ we need

$$\frac{1}{y'(z_s)} = 0$$

This means the surface must meet the brane perpendicularly

Also since the action is independent of y, we immediately get a first integral of motion

$$y' = -\left(\frac{z}{z_s}\right)^{d-1} \sqrt{\frac{1}{h(z)\left[1-\left(\frac{z}{z_s}\right)^{d-1}\right]}}$$

Notice this automatically satisfies the boundary condition at the brane.

We now find:

$$A = 2 \lim_{\varepsilon \to 0} \left[\frac{-1}{(d-2)\varepsilon^{d-2}} + \int_{\varepsilon}^{z_*} \frac{dz}{z^{d-1}} \sqrt{\frac{1}{h(z)} + y'^2} \right]$$

Note that

a) We have inserted a factor of 2 since there are two regions (on the two asymptotic boundaries)

b) We have used the same UV regulator that we used for the

$y = \text{const.}$ surface

3) The <u>difference</u> of the areas is independent of the UV regulator

We can also determine the amount the surface travels in y as

$$y_0 = \int_0^{z_s} dz \, \frac{1}{\sqrt{h(z)}} \, \frac{(z/z_s)^{d-1}}{\sqrt{1-\left(\frac{z}{z_s}\right)^{d-1}}}$$

which is finite

Now, let us consider how the entanglement entropy varies with time.

First, we need to ask which surface wins at $t=0$.

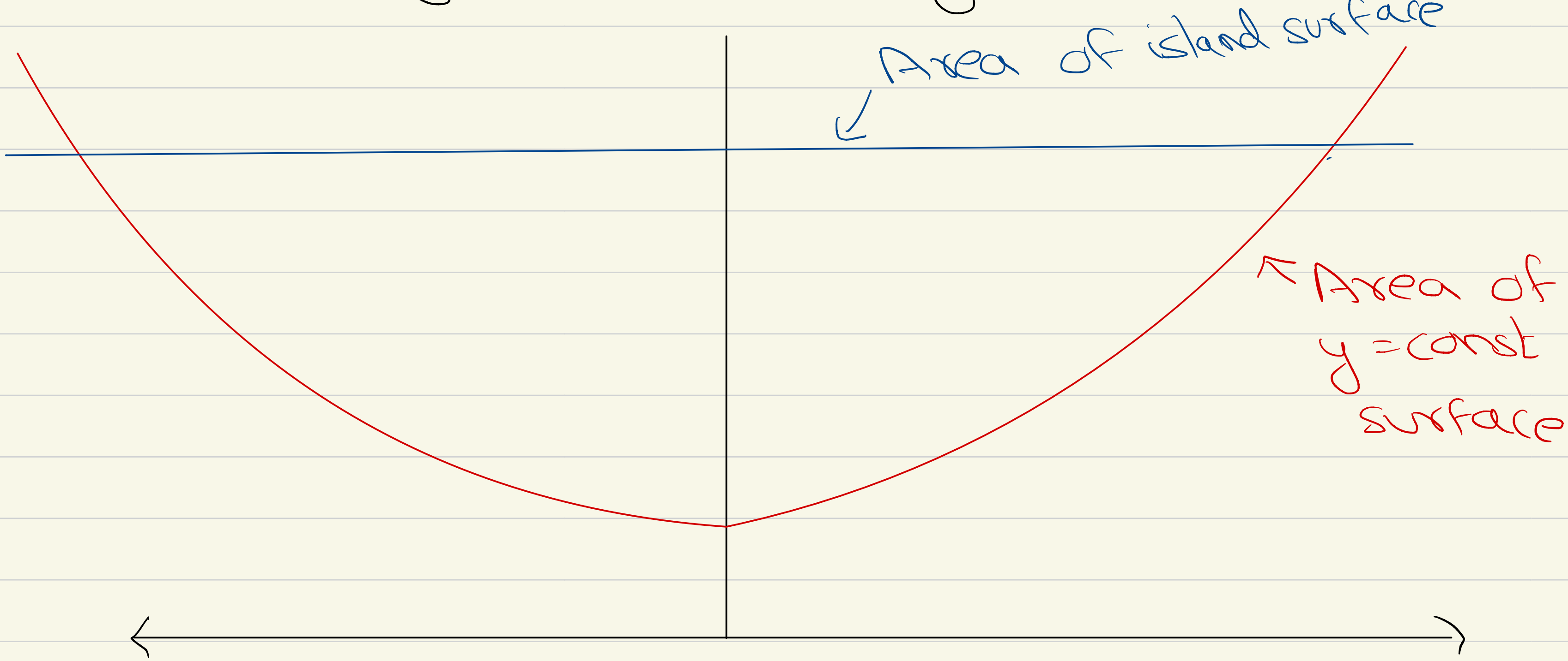
This is given by comparing

$$\Delta A = \int_0^{z_s} dz \frac{1}{z^{d-1}\sqrt{h(z)}} \frac{1}{\sqrt{1-\left(\frac{z}{z_s}\right)^{d-1}}} - \int_0^{z_0} dz \frac{1}{z^{d-1}\sqrt{h(z)}}$$

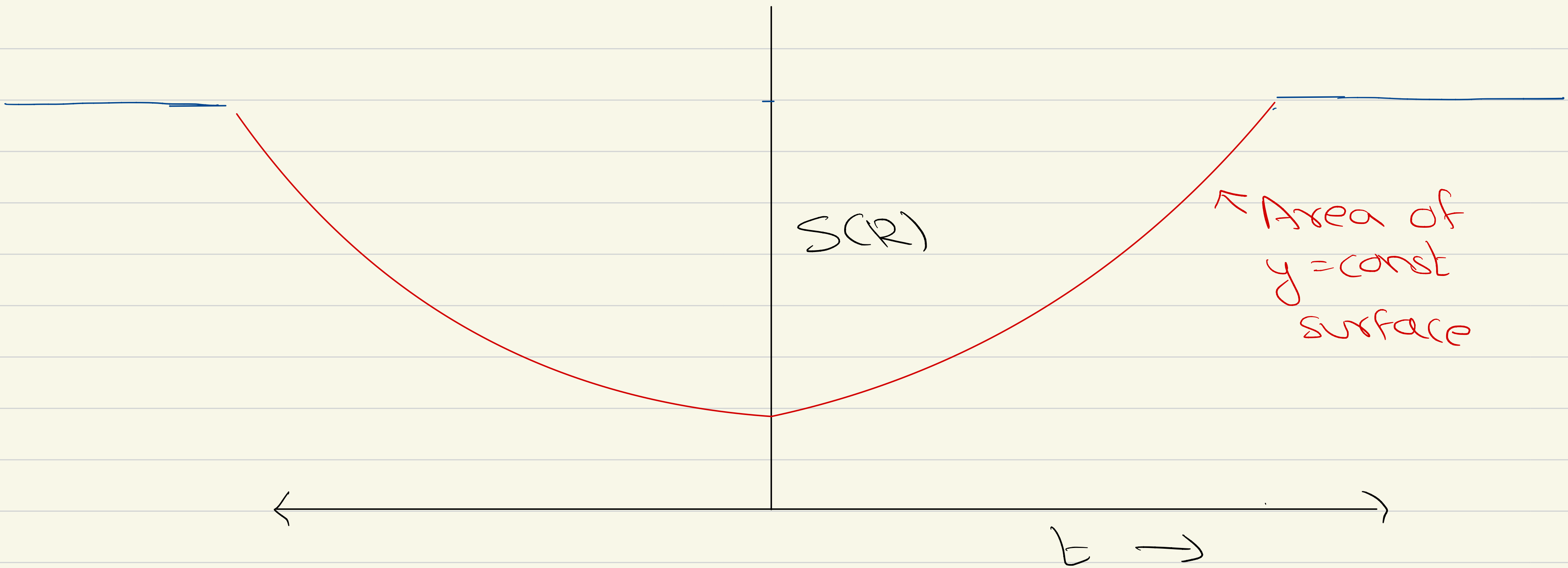
Say $\Delta A > 0$ at $t > 0$.

Note no regulator

Then initially $S(R)$ grows with time



We are instructed to pick the surface with **minimum area**.

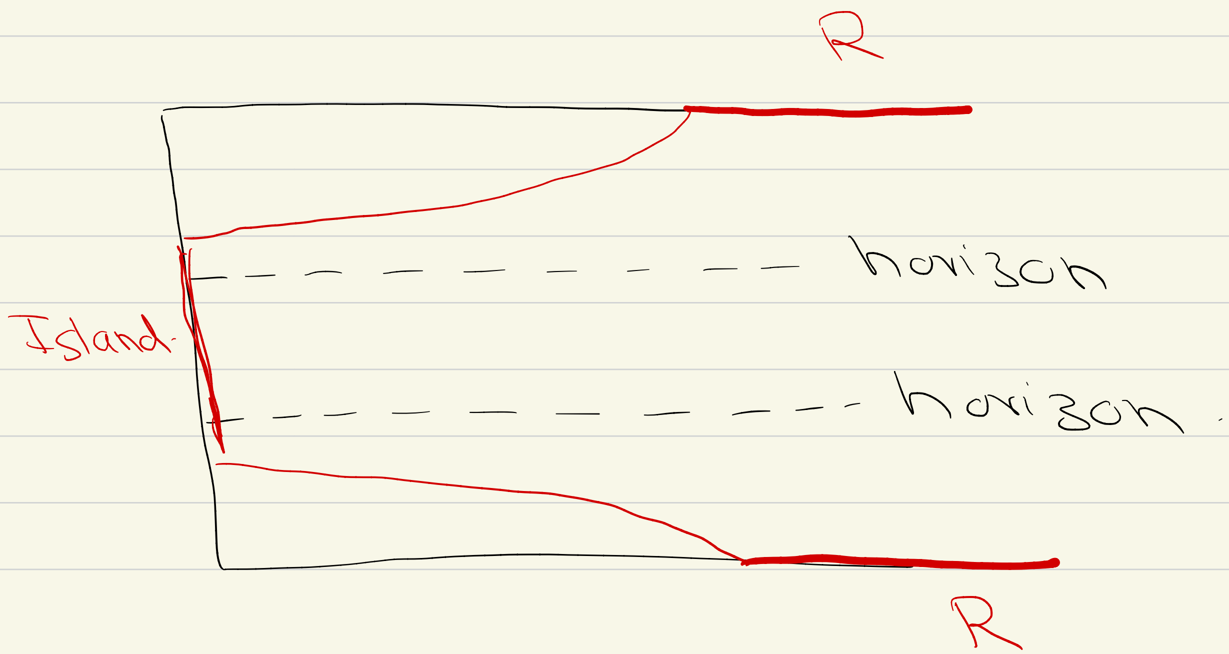


Therefore we get the Page curve.

The value of the "Page time" and also whether there is a transition at all depends on y_0 being large enough

This is related to islands.

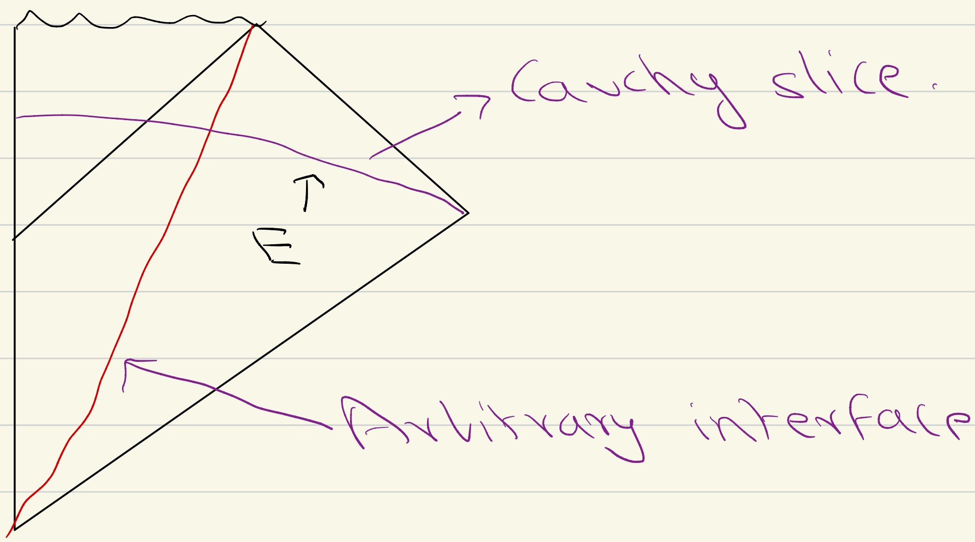
We find that for late enough times the entanglement wedge of the boundary region includes a piece of the brane



So we see that islands can help resolve puzzles that emerge in the holographic computation of entropy.

But it is important that all this works to answer a **nongravitational question.**

There is some sloppiness in the literature as well where we might have diagrams like this:



But this is not the setting for any precise calculation!

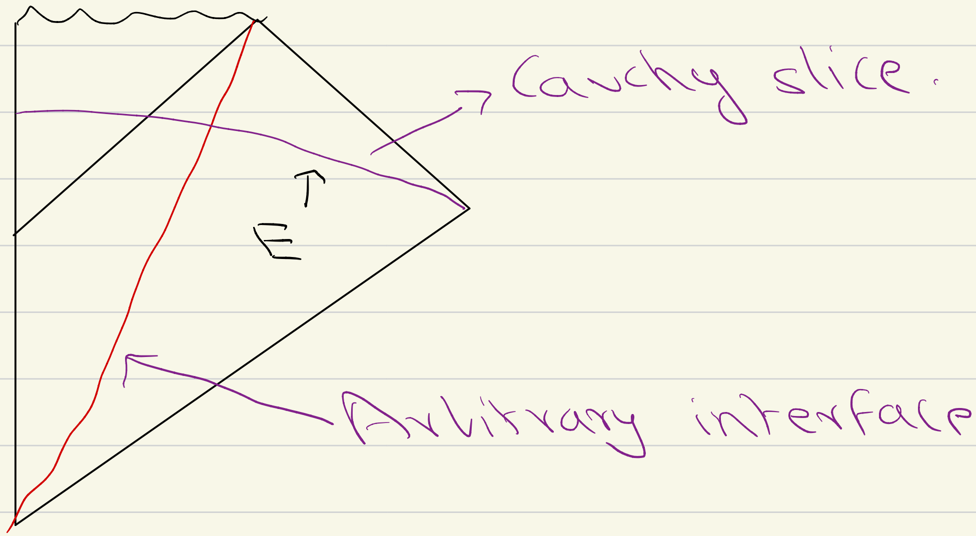
so some authors use

$$S(E) = \min \left[\operatorname{ext} \left[\frac{A(\partial \text{island})}{4G} + S_{\text{bulk}}(E \cup \text{island}) \right] \right]$$

even when E is in a region with dynamical gravity

But this is not justified by any clear calculation.

There is also clearly some tension with the principle of holography of information



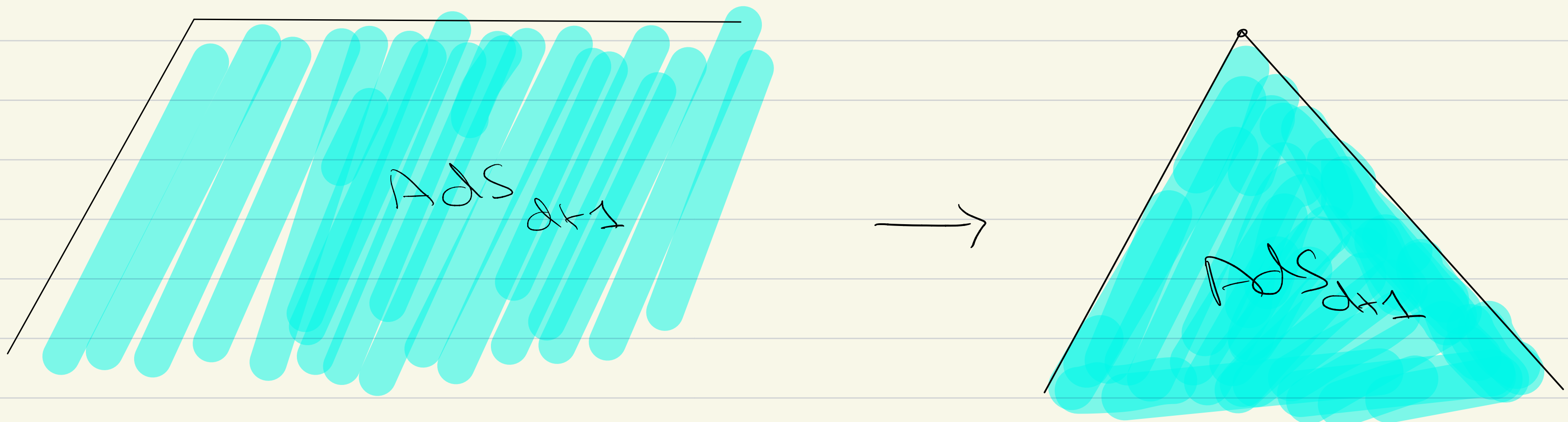
The principle of holography of information would imply that E always has information about the interior.

So SCE1 is flat!

In fact, there is a way to bring these discussions closer.

Even within the braneworld framework, one can turn on **gravity in the bath!**

The idea is to "push the boundary into the bulk"



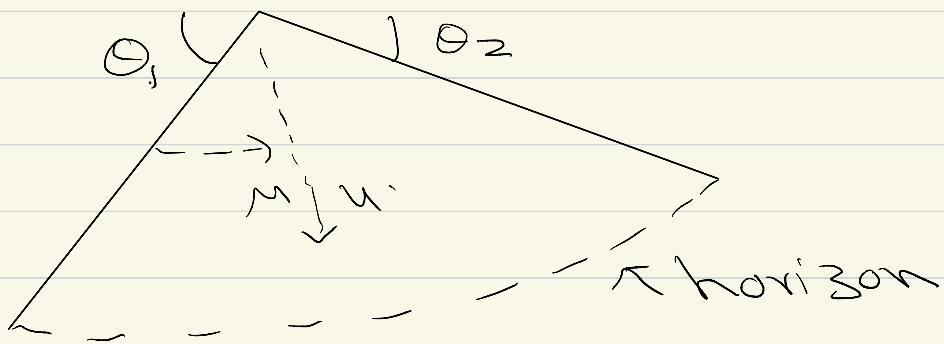
This can be made precise in the following setting.

We consider the black string metric

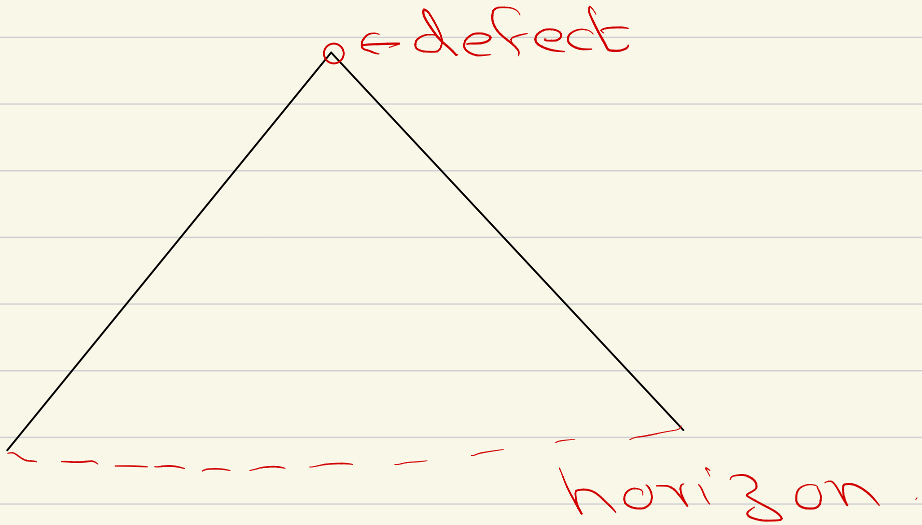
$$ds^2 = \frac{1}{u^2 \sin^2 \mu} \left[-h(u) dt^2 + \frac{du^2}{h(u)} + d\vec{x}^2 + u^2 d\mu^2 \right]$$

(d-2)-dim.

terminated by branes at θ_1 and θ_2.



The boundary is now gone and only is only left with a non-gravitating defect. The defect has dimension $d-1$.



[There is another asymptotic region on the other side.]

This geometry is nice enough so that we can put branes at any angle provided we choose the right tension.

So we can now look for minimal surfaces that end on both branes.

The new rule is:

"The surface should be an extremum at both end-points."

The justification is that now since the bath has dynamical gravity, it should not be treated differently from the system.

We are looking for surfaces like this extremized w.r.t. P_1 and P_2

The result is that the only such surface is the <u>horizon</u>

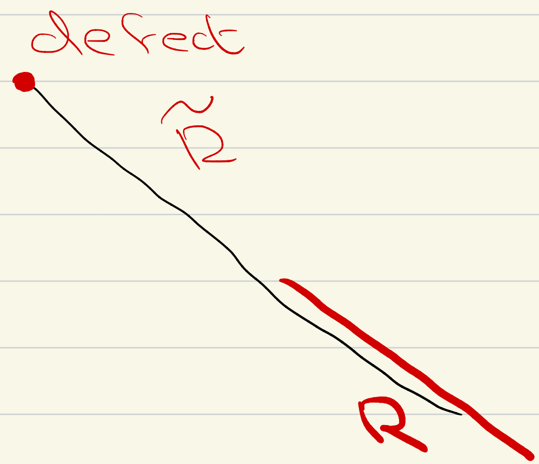
a) So it is <u>not</u> <u>possible</u> to define a non-trivial radiation region.

b) the entanglement wedge of the defect is always the entire exterior.

c) There is <u>no</u> <u>Page</u> <u>Curve</u>

So we find results that are **consistent** with the principle of holography of information,

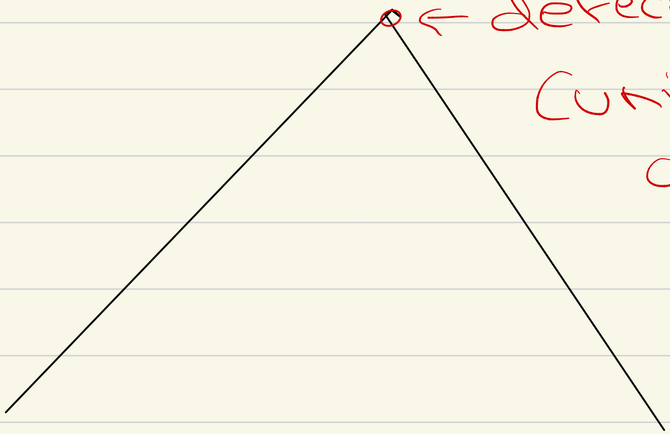
with dynamical gravity, we do <u>not</u> <u>expect</u> the algebra on the bath to factorize into R and $\tilde{R}$



These points can all be understood from our previous discussions

a) Holography of information $\Rightarrow$ no gauge-invariant operators localized purely in R.
[Wilson lines have to run through $\tilde{R}$]

b) More generally, holography of information implies that all info about the geometry is available in the defect



← defect has info about exterior

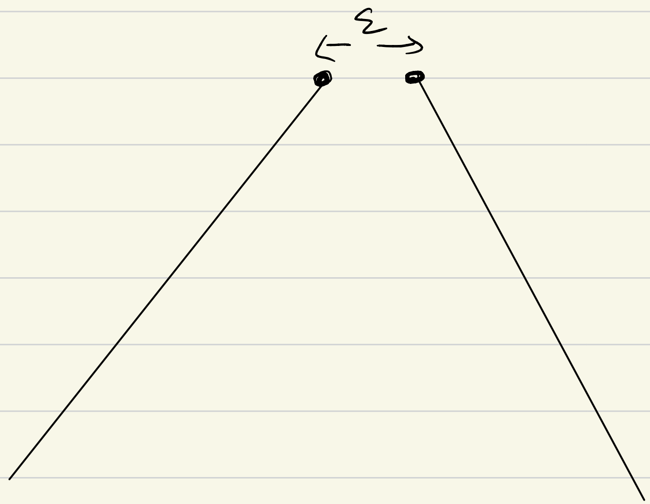
(union of defects on left and right has info about the entire geometry).

This is also consistent with the idea of "wedge holography", which states that gravity on a AdS_{d+1} wedge is dual to a CFT_{d-1}.

On the other hand, we can still ask other questions whose answer is the Page curve.

In this setup, it is possible to **internally divide** the degrees of freedom on the defect.

We can think of this geometry as being obtained from the limit of geometries like this

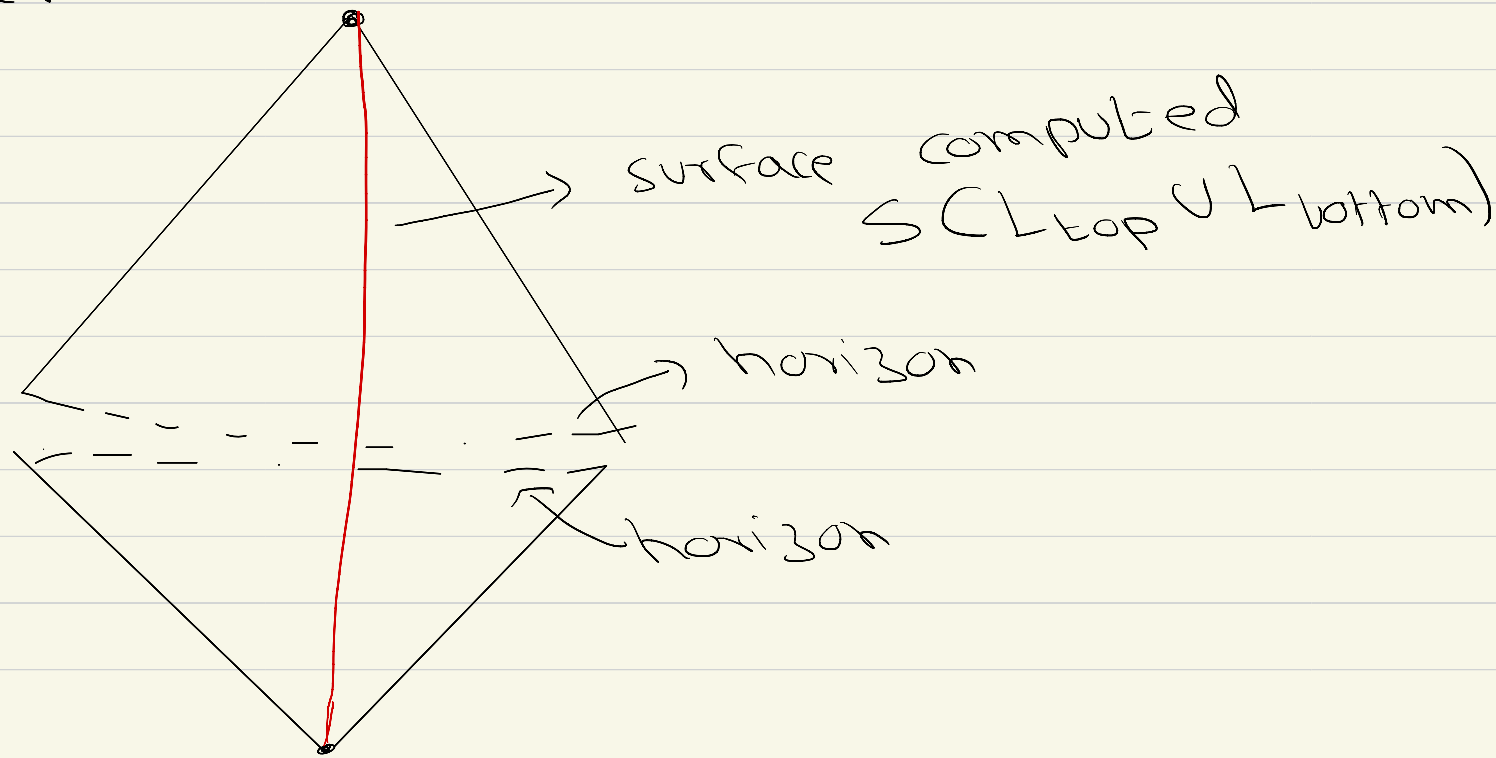


So one can think of a "left" CFT_{d-1} and a "right" CFT_{d-1}.

One can then ask about

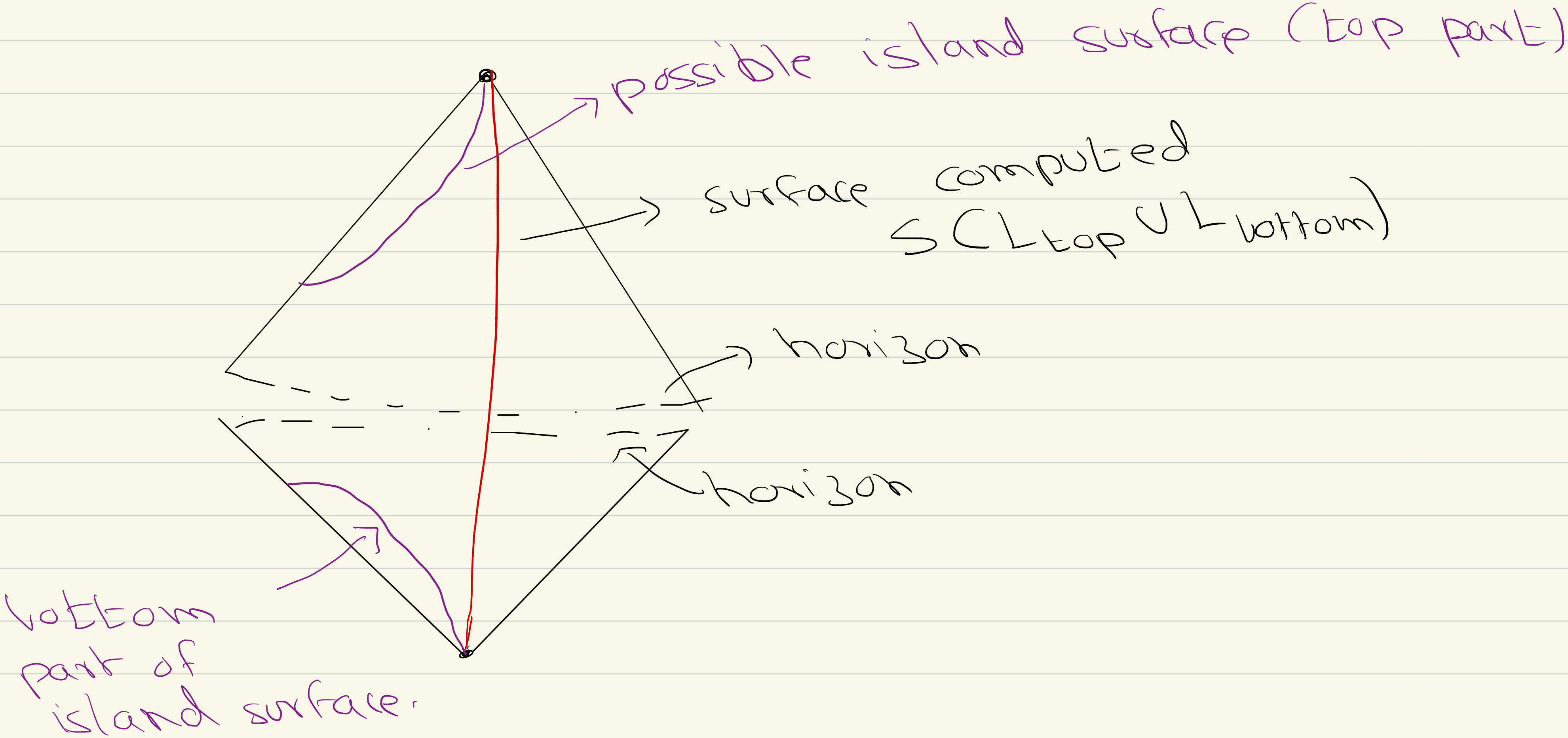
S_{LR} ← again a nongravitational division.

which is holographically computed by surfaces which run from the defect to one brane. or one defect to another



with time, the area of this surface grows.

So after some time, an island surface wins.



Punchline: when gravity is dynamical, there is no Page curve for the radiation.

But Page curve may still answer other interesting questions!

Motivated by this philosophy we can ask:

"Is the Page curve relevant for b.h. evaporation in flat space."

A reminder

Fine-grained entropy of this segment is constant (as fn of u_0) since information is always at $\mathcal{I}^+$.

Can we restrict the operator algebra in some way to neglect gravitational effects and obtain a Page curve?

The degrees of freedom of the metric are divided into:

the Bondi mass aspect $m(u,\Omega)$ and the Bondi news

$$N_{AB}(u,\Omega)$$

Usually N_{AB} is the "dynamical graviton" and m is the constrained piece.

The commutators are

$$[N_{AB}(u,\Omega), N_{CD}(u',\Omega')] = i16\pi G\, \partial_u \delta(u-u') \times \hat{\delta}(\Omega,\Omega') \times [\gamma_{A(C}\gamma_{D)B} - \tfrac{1}{2}\gamma_{AB}\gamma_{CD}]$$

The lack of factorization comes from the constraints:

$$\left[\int m(u,\Omega)\, d^2\Omega \,,\, N_{AB}(u',\Omega')\right] = 4\pi G\, i\, \partial_{u'} N_{AB}(u',\Omega')\, \Theta(u'-u)$$

lack of factorization.

So one natural guess is.

Keep only N_{AB} in algebra and drop m

1) This is a little unnatural physically since all components of the metric appear together in any physical measurement such as the Riemann tensor.

But it is mathematically consistent.

2) There is an additional subtlety.

Recall that the Hilbert space has soft sectors.

$$|\underset{\text{hard}}{n}, \{\underset{\text{soft}}{s}\}\rangle$$

The news operators are **blind** to the soft-sector.

So if hard-soft entanglement is high, no Page curve.

<u>Conclusion</u>: **<u>may</u>** be possible to obtain a Page curve in flat space by neglecting constrained components of the metric, but this depends on degree of hard-soft entanglement in black hole states.