1 April 2021

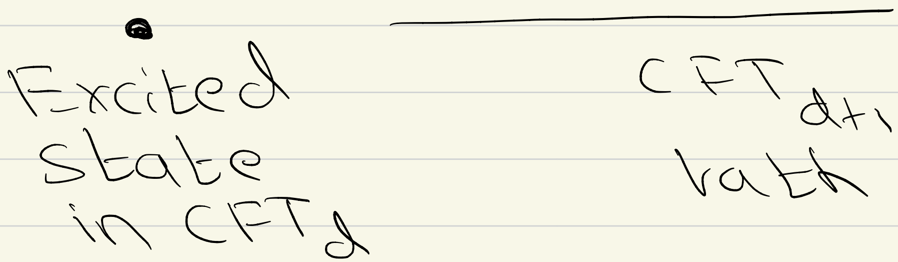
Lecture 22: Paradoxes with the eternal black hole

Before we start, we will revisit a question from yesterday.

Q) If we couple a CFT state dual to a large bh to a bath, we expect to see a Page Curve. What does this tell us about the interior?

Ans) Nothing!

As we emphasized, the Page curve can be understood as a nongravitational question.



we are guaranteed to see the Page curve.

One may ask: if the geometry has a firewall, how will the dual holographic computation work?

Ans) We don't know! Since no one has written a firewall "metric"

We will have to generalize the holographic EE prescription in some way.

Yesterday we reviewed the eternal black hole and said we would find a paradox with the following 3 assumptions:

1) Eternal black hole is dual to the thermofield double state

2) Dof in the interior are described by the same operators in the eternal b.h. and a class of states related to the eternal b.h. by Hamiltonian time evolution

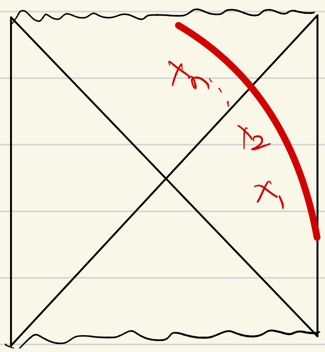
3) Disentangled states $|E, E\rangle$ are not connected by a wormhole.

[Written a little confusingly in the review at the moment. See arXiv: 1503.08825 or arXiv: 1502.06692]

Consider the experience of an infalling observer who jumps in from the right boundary.

The experience of this observer is described by some correlators measured along the observer's trajectory.

$$\langle \psi_{tfd} | \phi(x_1) \ldots \phi(x_n) | \psi_{tfd} \rangle .$$



We are only interested in the exterior and the region just behind the horizon.

Now consider translating the entire trajectory by time τ.

Since there is no **natural origin of time**, we expect that

$$\langle \psi_{tfd} | e^{iH_R\tau} \phi(x_1) \ldots \phi(x_n) e^{-iH_R\tau} | \psi \rangle_{tfd}$$

$$= \langle \psi_{tfd} | \phi(x_1) \ldots \phi(x_n) | \psi_{tfd} \rangle$$

Moreover this should be true for arbitrary $\tau \in (-\infty, \infty)$

[This is also clear from the geometry. Explain because the Penrose diagram may be confusing.]

But now we can write

$$\langle \psi_{tfd} | e^{iH_R\tau} \phi(x_1) \ldots \phi(x_n) e^{-iH_R\tau} | \psi_{tfd} \rangle$$

$$= \frac{1}{Z(\beta)} \sum_E e^{-\beta E} \langle E, E | \phi(x_1) \ldots \phi(x_n) | E, E \rangle$$

$$+ \frac{1}{Z(\beta)} \sum_{E \neq E'} e^{-\beta\left(\frac{E+E'}{2}\right)} e^{i(E'-E)\tau}$$

$$\langle E', E' | \phi(x_1) \ldots \phi(x_n) | E, E \rangle .$$

The key point is that this has to be independent of τ for arbitrary τ.

This can only happen if the second term vanishes.

$$\langle E', E' | \phi(x_1) \ldots \phi(x_n) | E, E \rangle = 0$$

for $E' \neq E$

So we expect that the experience of the infalling observer is described by

$$\frac{1}{Z(\beta)} \sum e^{-\beta E} \langle E, E | \phi(x_1) \ldots \phi(x_n) | E, E \rangle$$

Now, by the usual argument of equivalence of ensembles,

$$\frac{1}{Z(\beta)} \sum_E e^{-\beta E} \langle E, E | A | E, E \rangle$$

$$= \langle \tilde{E}, \tilde{E} | A | \tilde{E}, \tilde{E} \rangle$$

where $|\tilde{E}\rangle$ is a **typical** energy eigenstate from the band of energies relevant at temperature β^{-1}.

So if we have operators $\phi(x_i)$ that can be used in all time-shifted states, they also yield the experience of the observer is the **disentangled state.**

Now let us invoke the "no wormhole in disentangled states" assumption

This means that in the state $|E,E\rangle$ no unitary on the left can affect the right infalling observer

so

$$\langle E,E| U_L^\dagger \phi(x_1) \dots \phi(x_n) U_L |E,E\rangle$$

$$= \langle E,E| \phi(x_1) \dots \phi(x_n) |E,E\rangle$$

for any unitary operator U_L.

If this is true for any unitary V_L then $\phi(x_i)$ must be operators only in the right CFT.

But now we know there are no operators in purely the right CFT that can describe a smooth experience for the infalling observer in typical energy eigenstates.

So using the same paradoxes as we used in the single-sided black hole, we now find a paradox for the eternal b.h.

Recap of logic

If ops "work" in TFD + cousins $\longrightarrow$ must also "work" in disentangled states

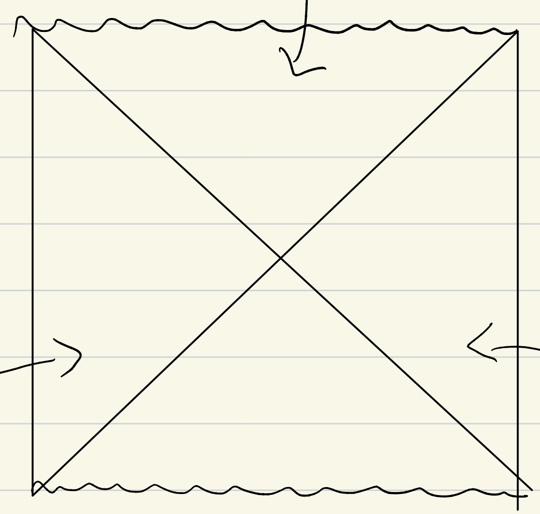
If ops work in disentangled states $\xrightarrow[\text{in disentangled states}]{\text{no wormhole}}$ must also work in a single CFT

But previous paradoxes apply here!

Some elaboration

Before these arguments, and even now sometimes, we find the argument that the reconstruction in the eternal b.h. should be as follows

$$\phi = \sum a^R_\omega g^1(t,r,\Omega) + a^L_\omega g^2(t,r,\Omega)$$



$$\phi = \sum a^L_\omega f_\omega(t,r,\Omega) + h.c.$$

$$\phi = \sum a^R_\omega f_\omega(t,r,\Omega) + h.c.$$

i.e. a common proposal is:

$$\text{set } \tilde{a}_\omega = a^L_\omega$$

$a^{L/R}_\omega \leftarrow$ modes from the left/right CFT.

Indeed it is true that

$$\langle \Psi_{tfd} | \, a^L_\omega \, a^R_\omega \, | \Psi_{tfd} \rangle$$

$$= \frac{e^{-\beta\omega/2}}{1 - e^{-\beta\omega}}$$

But these operators do **not** yield the right correlators in the time-shifted states.

$$\langle \psi_{tfd} | e^{-iH_R\tau} a^L_\omega \, a^R_\omega \, e^{iH_R\tau} | \psi_{tfd} \rangle$$

$$= e^{i\omega\tau} \langle \psi_{tfd} | a^L_\omega \, a^R_\omega | \psi_{tfd} \rangle$$

So the use of a^L_ω as $\tilde{a}_\omega$ would suggest that the state

$$e^{iH_R\tau} | \psi_{tfd} \rangle$$

is singular at the horizon. But this is unphysical.

Second, note that if we allow ourselves to violate the "no wormhole" condition, it is possible to write down operators that work correctly in $|\Psi_{tfd}\rangle$ and also in the time-shifted states.

See 1503.08825 ; pg 64.

Paradoxes about exponential decay

There are other kinds of paradoxes for large bhs which are not conceptually puzzling, but are amenable to calculations.

Consider the two-point function of a boundary operator in a typical pure state

$$\langle \psi | O_R(t) O_R(0) | \psi \rangle$$

Here we consider a boundary operator dual to some propagating scalar field.

So this can also be computed using WdR methods.

For simplicity, we set

$$\langle E | O(0) | E \rangle = 0$$

[one-pt fn is 0]

If we do a bulk computation, we find this decays indefinitely as

$$e^{-\kappa \tau}$$

where κ is controlled by the **quasinormal modes** of the b.h.

But let us see what we expect from the CFT perspective at late times.

$$\langle \psi | O(t) O(0) | \psi \rangle = O(e^{-S/2}) \quad \text{at late times}$$

[we are looking at overlap of two random vectors in a e^S dim space.]

We see that at

$$t = O(S)$$

we find a contradiction between the bulk result and the expectation from unitarity.

This is a nonperturbative correction.

It may even come from the other saddle (gas of gravitons) that we have neglected.

A similar issue arises with the analytically continued partition function or spectral form factor.

We considered previously a set of time-shifted thermofield doubled states

$$|\psi_\tau\rangle = e^{-iH_R\tau} |\psi_{tfd}\rangle$$

What is the overlap with the original state?

$$\langle \psi_\tau | \psi_{tfd}\rangle = \frac{Z(\beta + i\tau)}{Z(\beta)}$$

For short times, we can compute.

$$|\langle \psi_\tau | \psi_{tfd} \rangle|^2 = e^{-\tau^2 C/\beta^2}$$

At late times we have.

$$\frac{\sum e^{-\beta E} e^{-iE\tau}}{\sum e^{-\beta E}} \sim O(e^{-S(2)})$$

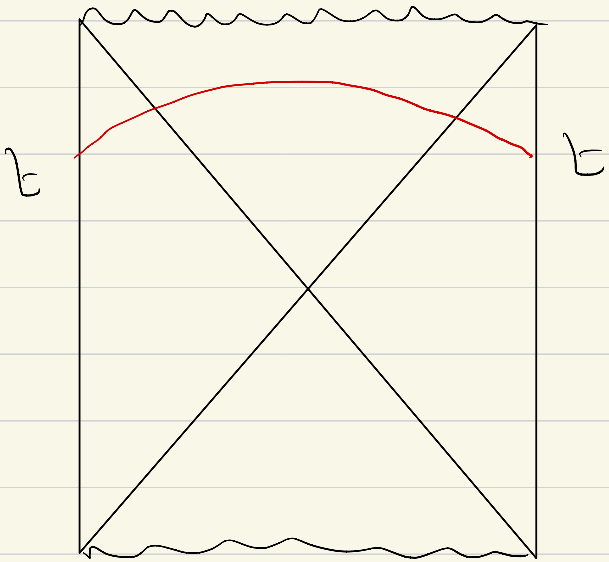
Since e^S terms contribute to both sums but we have random phases in the numerator.

In some cases, this can be computed, and turns out to have a rich structure.

Bags of gold

We can also set up a simplified version of a paradox called the "bags of gold" paradox

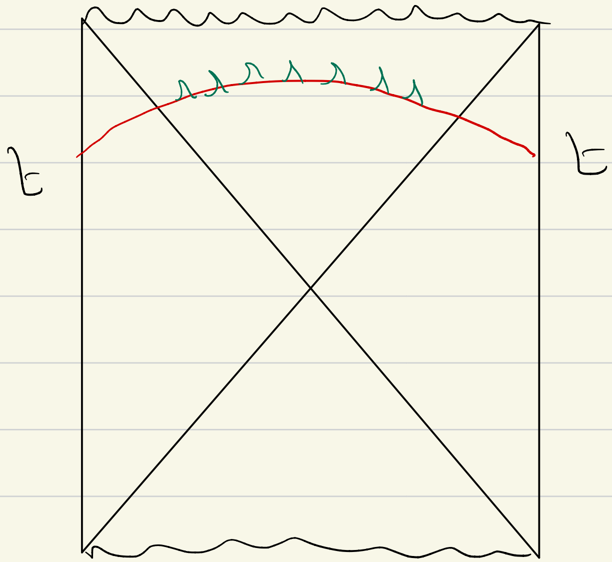
As we discussed the volume of max-volume slices grows at large times



$$\frac{dV}{dt} \propto t$$

Now say we try and count the entropy using naive techniques

we can try and put low-energy excitations on the slice while removing energy from the background.



If we think of these as a gas then

$S_{gas} \propto V$

so at late times

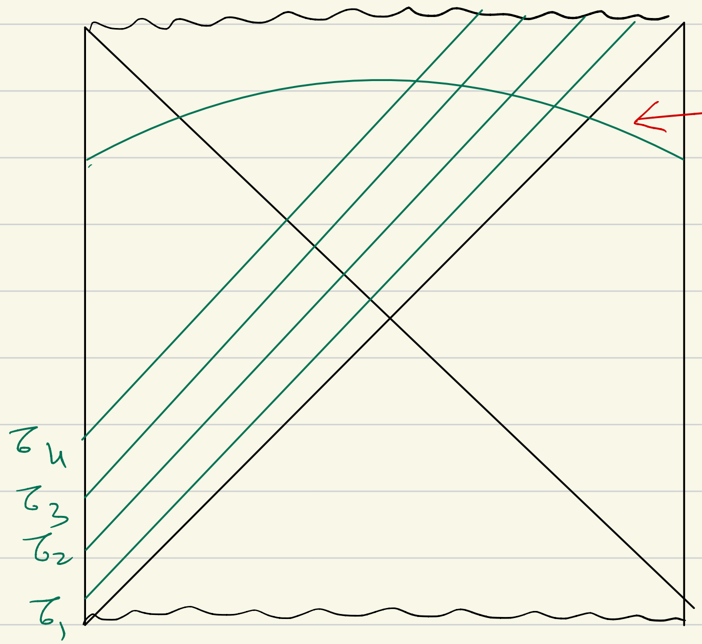
$$S_{gas} \gg S_{bh}.$$

This is similar to the paradox we considered while studying islands.

Here we can try and understand the resolution **more directly.**

We can try and "create" this dilute gas of excitations directly

$$U_L(\tau_1) \dots U_L(\tau_n) |\psi_{tfd}\rangle.$$



looks like a dilute gas of excitations if $|\tau_i - \tau_j|$ is large.

But the key point is that the inner-product between such states never gets driven to zero.

$$\langle \psi_{tfd} | U_L(\tau_1) \, U_L(\tau_2) | \psi_{tfd} \rangle = O(e^{-S})$$

So even though these states might appear to be independent naively the inner-product has a fat-tail that tells us that we are overcounting if we count all these excitations independently.