15 April 2021

Lecture 26: Discussion

Yesterday we pointed out that:

1) State-dependent interior constructions suggest an anamolous sensitivity of the bh interior to low energy excitations

$$\left| \langle \psi | U^\dagger A U | \psi \rangle - \langle \psi | A | \psi \rangle \right| \lesssim 2\sqrt{\beta \delta E\, 6}$$

2) But if we restrict

a) U to those that can be obtained through

$$H \to H + J(t)\, Q(t)$$

and

ii) observables A to a causal patch

$$A = \phi(x_1) \ldots \phi(x_n) \quad \text{[All } x_i \text{ are in a causal patch]}$$

then the inequality is obeyed.

<u>Unanswered questions</u>

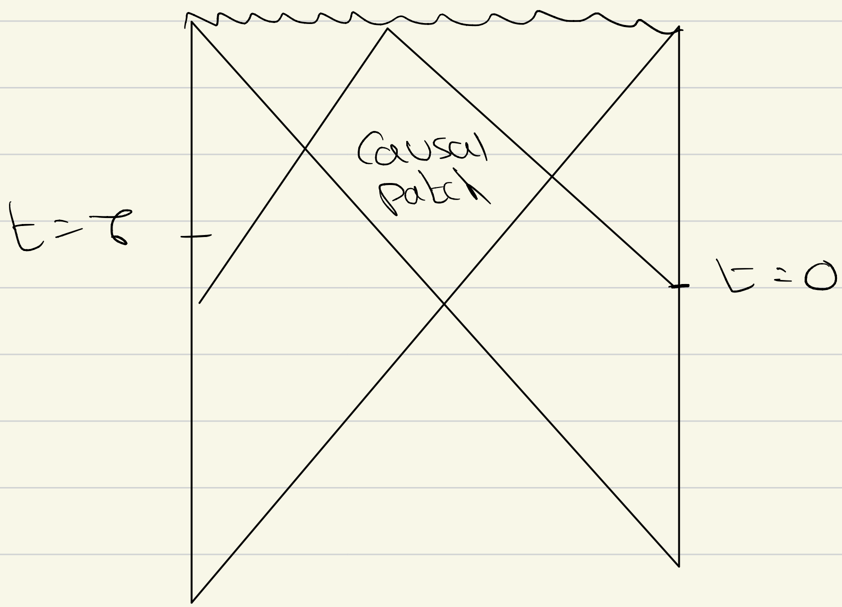
1) From the point of view of bulk EFT, what is wrong with

$$e^{i\theta N_w}$$

We often use such operators in our analysis even if such deformations are difficult to generate.

2) We also often consider observables outside a causal patch.

eg. in the eternal black hole consider a point at $t=0$ on the right boundary and $t=\tau>0$ on the left boundary.



The theory gives a prediction for

$$\langle \Psi | O_L(\tau) \, O_R(0) | \Psi \rangle$$

even though this is unobservable within simple bulk experiments.

So we have a few options

1) Perhaps the "low-energy excitation bound" just does not hold for excitations of the form $e^{i\theta N w}$ and correlators outside of a causal patch.

This is vaguely unsatisfying. (a)QFT allows to consider observables beyond those that are observable in a simple way

(1) We need to ensure that subtleties with locality in gravity do not spoil this picture.

2) Maybe naive EFT does not correctly describe the response of the bulk to excitations like

$$e^{i\theta N_w}$$

If so, why?

What causes large backreaction?

3) Maybe this paradox rules out state-dep operators.

In this case, the mirror operator construction still applies to those microstates that have a smooth interior but not to typical microstates.

There has been a lot of debate on state dependence. Some people say it "violates QM" so must be wrong. Others say it must be obviously true!

But this specific puzzle (also called the "Born rule" paradox) appears to be the key to clearing up this issue!

This is not widely recognized and hopefully a reader of this will help in clearing up the issue completely!

Other interior constructions

One might wonder if alternative constructions of the interior might avoid this problem.

But the mirror operator construction is dictated by effective field theory.

So any other construction, if correct, must coincide with this construction for states with a smooth interior at least on H_ψ.

A number of alternative constructions have been proposed.

The following questions are useful if one encounters a proposal for the interior.

1) Small commutator with exterior operators?

$$[\tilde{a}_\omega , a_\omega] \doteq 0$$ [within simple correlators]

2) right commutator with the Hamiltonian?

$$[H, \tilde{a}_\omega] \doteq \omega \tilde{a}_\omega$$

3) Frozen vacuum?

$$\tilde{a}_\omega A |\psi_{ne}\rangle = A \, U a_\omega^{+} U^{+} |\psi_{ne}\rangle e^{-\beta\omega/2}$$

4) Generic states?

How does the proposed operator act on generic states?

Recall that if we give up the demand that "generic states have smooth horizons" then we can use state-ind. mirror operators.

Some proposals are made only for the thermofield doubled state in which case they do not shed light on the physics of typical states.

So other constructions can differ in

a) How they treat generic states

b) How they act <u>outside</u> H_ψ {which is not specified for mirror operators.}

State dependence elsewhere

State dependence appears to crop up in other places in AdS/CFT.

The simplest case is the Ryu-Takayanagi formula.

We have

$$S = \frac{A}{4G_N}$$

On the LHS, "A" seems to be an observable in the bulk.

But on the R.H.S., S is <u>not</u> the expectation value of any observable.

<u>Proof</u>

Let $H = H_1 \otimes H_2$

then $S_2 \neq \langle X \rangle$ for any X.

Assume that $S_2 = \langle X \rangle$

Then $X \geq 0$ since $S_2 \geq 0$

Now consider a basis for $H_1 \otimes H_2$

$$|i,j\rangle$$

we have

$$S_2(|i,j\rangle) = 0$$

$$\Rightarrow \quad \langle i,j| X |i,j\rangle = 0$$

so X is 0 for all elements of a basis and since all its eigenvalues must be non-negative, we find

$$X = 0!$$

which is absurd. So X does not exist.

For small combinations of states in a big Hilbert space, S^0 acts approximately linearly.

But it has not been investigated whether this state dependence can have significant observable effects.

Do typical states have structure

Let us review the discussion that we have been having for a few lectures.

About a large black hole geometry, assuming a smooth horizon, we can use QFT in curved spacetime to compute some correlators

$$G(x_1 \ldots x_n) = \langle \phi(x_1) \ldots \phi(x_n) \rangle_{QFT}$$

The question we have been asking so far is as follows.

Do there exist operators $\phi_{CFT}(x_i)$ so that in a typical state, $|\psi\rangle$

$$\langle\psi|\phi_{CFT}(x_1)\ldots\phi_{CFT}(x_n)|\psi\rangle = G(x_1\ldots x_n)$$

This is subtly different from the question of
"Are $\phi_{CFT}(x_i)$ the "right" bulk operators?"

Q above is only an existence question.

The answer we find is:

Yes, if we allow state-dependent mirror operators.

No, otherwise.

Eternal b.h

If we give up state dependence, we must also give up the idea of a duality between the thermofield double state and the eternal black hole.

So we are not allowed to say:

"generic states for single-sided black holes have firewalls but eternal b.h. has a smooth interior"

We need to somehow modify the eternal b.h also.

Flat space black holes

Recall that flat space black holes are exponentially atypical states

This is because

$$S_{bh} < S_{hawking\ rad}$$

which is why the black hole evaporates in the first place.

For such states the paradoxes we saw for large black holes do not appear.

Said another way.

Say $|\psi_1\rangle$ is a flat space b.h. microstate.
We can use the mirror operator construction
to generate

$$\tilde{a}_\omega^{\psi_1}$$

on $\mathcal{H}_{\psi_1}$

Similarly about other microstates $|\psi_2\rangle \dots |\psi_{e^S}\rangle$
we can construct mirrors

$$\tilde{a}_\omega^{\psi_i}$$

In flat space the operator

$$\tilde{a}_\omega = \sum_{i=1}^{e^S} \tilde{a}_\omega^{\psi_i}$$

may give a state-independent description of the interior.

For large AdS black holes we argued that even about $|\psi_1\rangle$

$$\langle \psi_1 | A_i \left(\tilde{a}_\omega^{\psi_2} + \dots \tilde{a}_\omega^{\psi_{e^S}} \right) A_j | \psi \rangle$$

may give an appreciable contribution since the large number of terms may compensate for the small expected size of each term.

But in flat space the no. of terms is still e^S but the size of each cross-term is set by $e^{-S_{rad}}$.

This is <u>not</u> a watertight argument.

But it is accurate that

"No currently unresolved paradox suggests that flat space b.h. or small b.h. in AdS have firewalls / fuzzballs."

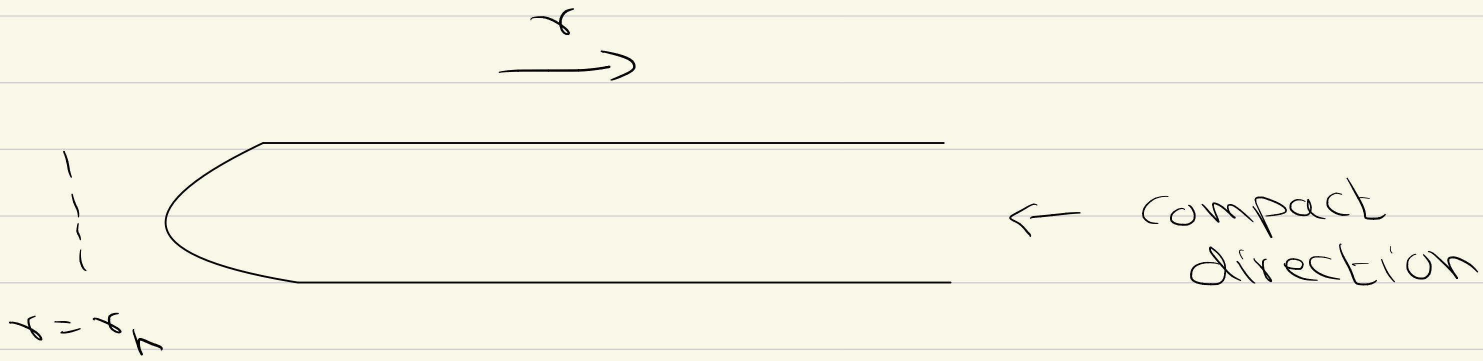
[The monogamy paradox resolved by the holography of info.]

We can also ask "do classical solutions tell us about structure at the horizon of typical states?"

A number of classical solutions called microstate geometries have been found with the same charges as black holes.

These solutions tend to differ from the conventional geometry even outside the horizon.

They avoid the no hair theorem by taking advantage of a compact direction that pinches off before the horizon is reached.



A number of such solutions have been found

But if one states that typical states have structure there are constraints from statistical mechanics

The constraints are as follows. First recall that typical states are exponentially close to the microcanonical ensemble and so they are exponentially close to each other.

For any projector P we have

$$\int d\mu_\psi \left(\langle\psi|P|\psi\rangle - \langle P\rangle \right)^2 \leq \frac{1}{e^S+1}$$

This also means that if $|\psi_1\rangle, |\psi_2\rangle$ are typical

$$\langle\psi_1|P|\psi_1\rangle - \langle\psi_2|P|\psi_2\rangle \sim O(e^{-S/2})$$

So there must be a universal microstate geometry to replace the Schwarzschild geometry.

The different microstate geometries have distinct features but only one must be relevant for typical states.

Which one?

One idea is that typical states correspond to the conventional geometry but the microstate geometries give us a basis.

But basis elements are also subject to constraints.

Say we have an observable so that

$$\frac{\sigma}{\langle A \rangle} \longrightarrow O\left(\frac{1}{S}\right)$$

where

$$\sigma^2 = \langle A^2 \rangle - \langle A \rangle^2$$

then "most" basis elements $|f_i\rangle$ must satisfy

$$\langle f_i | A | f_i \rangle - \langle A \rangle = O\left(\frac{1}{S}\right)$$

See 1804.10616 for a more precise result.

$$\sigma^2 = \frac{1}{e^S} \sum_i \langle f_i | A^2 | f_i \rangle - \langle A \rangle^2$$

$$= \frac{1}{e^S} \Big[\sum_i \left(\langle f_i | A^2 | f_i \rangle - \langle f_i | A | f_i \rangle^2 \right) \leftarrow \text{positive}$$

$$+ \sum_i \left(\langle f_i | A | f_i \rangle - \langle A \rangle \right)^2 \Big]$$

$\uparrow$ positive

so if an $O(1)$ fraction of basis elements have

$$\frac{\langle f_i | A | f_i \rangle - \langle A \rangle}{\langle A \rangle} = O(1)$$

we cannot have $\dfrac{\sigma}{\langle A \rangle} = O\left(\dfrac{1}{S}\right)$

This is relevant for black holes because using the Euclidean theory, for simple observables made out of the metric g, we estimate

$$\frac{\sigma^2}{\langle g \rangle^2} = O\left(\frac{1}{S}\right)$$

This is just the statement that the geometry is classical. Fluctuations come with a factor of

$$\left(\frac{r_h}{l_{Pl}}\right)^{d-2} = O\left(\frac{1}{S}\right)$$

So most basis elements must also be very close to the standard geometry and cannot be macroscopically distinct.

A short summary

Information Paradox

Hawking's original paradox

Pure & thermal states are very close

Monogamy paradox; principle of ignorance

Holography of Information

Long time 2-pt fn. Spectral form factor

Exponentially small corrections

Interior of typical large AdS black holes

State dependence

AdS black holes coupled to nongravitational baths

Islands.