Quantum Aspects of Black Holes, Lec 22, 16 Nov 2016

Last time, we discussed the cloning and strong subadditivity paradoxes.

Today, we will explore a resolution of this paradox using black hole complementarity

The word complementarity is used by different people in different ways!

First, I will describe a paper by Susskind & Thorlacius [hep-th/9308100] - one of the early papers in the subject.

The original idea was that we could have two alternative descriptions of the black hole

1) description of the outside observer who, for all purposes can think of a "membrane" at the stretched horizon and unitary evolution
2) description of the infalling observer who sees a smooth horizon, and cannot verify unitarity since

he perishes in the singularity.

Later, we will explore a different formulation of complementarity involving non-factorization of the Hilbert space when we consider very high-point correlators.

Susskind et. al. approach

Susskind et. al. considered some gedanken experiments to show that no observer could detect complementarity without knowing about Planck scale physics

First consider the relatively simple question of whether the horizon is empty or has a membrane at

$$r = r_h + \frac{l_{Pl}^2}{r_h}$$

which ensures that the area of the stretched horizon is l_{Pl}^2 larger the event-horizon.

Now consider an observer who falls in and sends messages reporting the smoothness of the horizon to an observer at ∞.

The red-shift factor is

$$\omega_\infty = \sqrt{1 - \frac{r_h}{r}}\ \omega_{in}$$

At the stretched horizon this becomes

$$\omega_\infty = \left(1 - \frac{r_h}{r_h\left(1 + \frac{l_{Pl}^2}{r_h^2}\right)}\right)^{1/2} \omega_{in}$$

$$= \frac{l_{Pl}}{r_h} \omega_{in}$$

Recall the outside observer is in a bath of radiation at temperature $\frac{1}{r_h}$

To be able to read the message, the outside observer requires

$$\omega_{\infty} > \frac{1}{r_h}.$$

but then $\omega_{in} > \frac{1}{\ell_{Pl}}$

so the infalling observer has to use Planckian frequencies to communicate the absence of a membrane.

alternately we can think of an observer who goes close to the horizon, and then fires his rockets and escapes.

What is the acceleration required to reach the stretched horizon and escape?

Recall that with

$$r_* = r + 2m \ln \frac{|r-2m|}{2m}$$

$$U = -4M e^{(r_*-t)/4M}$$

$$V = 4M e^{(t+r_*)/4M}$$

the metric near the horizon looks like

$$ds^2 = -dU\,dV + (2m)^2 d\Omega^2$$

at $r = r_h + \frac{l_{Pl}^2}{r_h}$

we have

$$r_* = r_h\left(1 + \ln \frac{l_{Pl}^2}{r_h^2}\right)$$

so

$$UV = -4r_h^2 \, e^{(1+\ln l_{Pl}^2/r_h^2)}$$

$$= -4e \, r_h^2 \cdot l_{Pl}^2/r_h^2 = -4e \, l_{Pl}^2$$

So $\quad a \sim \dfrac{1}{l_{Pl}}$

So the rocket-based observer sees an Unruh effect with temperature

$$T \sim M_{Pl}.$$

which swamps out any observations he might want to take on the smoothness of the horizon.

Now, let us consider an experiment involving both the infalling and the exterior observer.

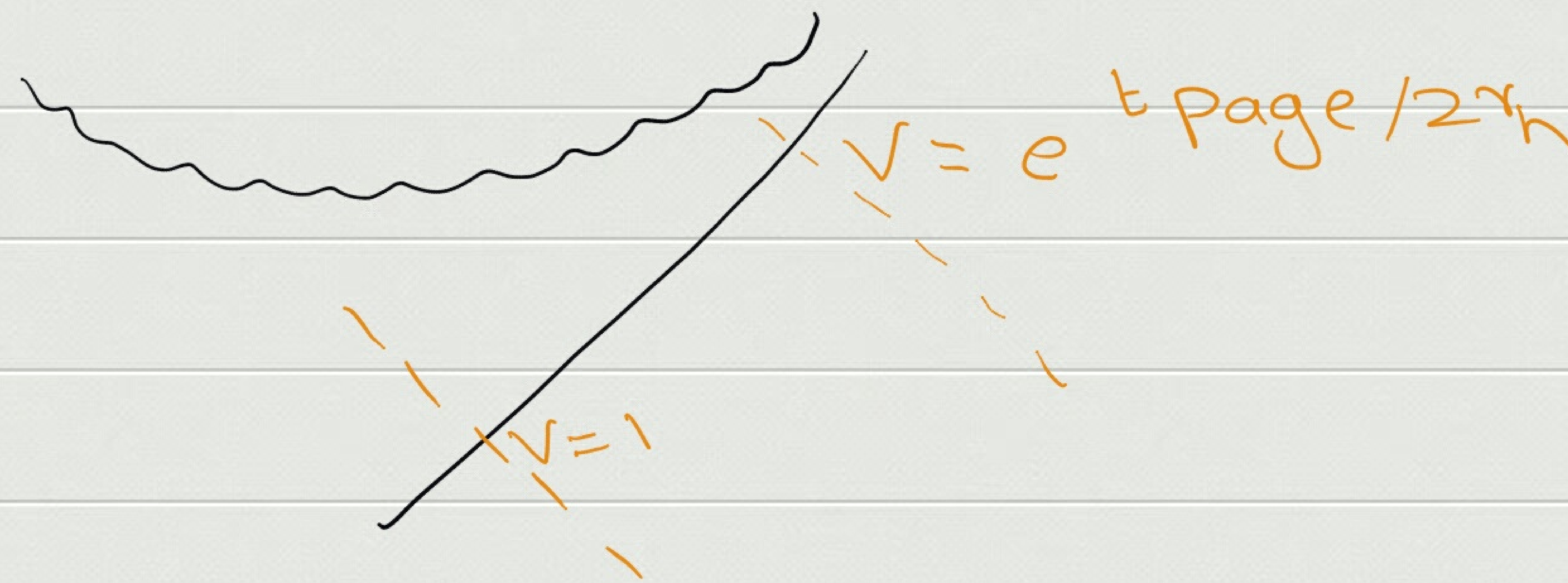
Let us see if the two observers together can detect quantum cloning

Say the first observer crosses the stretched horizon at $V=1$

The second observer crosses the horizon at $V = e^{r_h (r_h/l_{Pl}) K}$

[Recall $V = e^{(r_* + t)/2r_h}$]

where k is a numerical constant.



Recall that the singularity is at $UV=1$

Means the first observer has to send a message before he crosses

$$U = e^{-t_{page}/2r_h}$$

Near $V=1$, this corresponds to [Recall $UV = e^{r_*/r_h}$]

$$r_* = -t_{page}/2$$

which is $\frac{r_h - r}{r_h} = e^{-t_{page}/2r_h}$ or $r = r_h\left(1 - e^{-t_{page}/2r_h}\right)$.

The proper time to reach this r is $-\int_{r_h}^{r_h - \delta} dr \sqrt{1 - \frac{r_h}{r}}$

$$= r_h e^{-3t_{page}/4r_h} = r_h e^{-\#(r_h^2/l_p^2)}$$

numerical constant

In fact as long as the later observer jumps in after a time

$$r_h \log \frac{r_h}{l_{Pl}}$$

the first observer has to use Planckian frequencies.

Hayden & Preskill [0708.4025] argued that if someone collected all the radiation till Page time and then threw in new information, the black hole would emit it in

$$r_h \log r_h / l_{Pl}$$

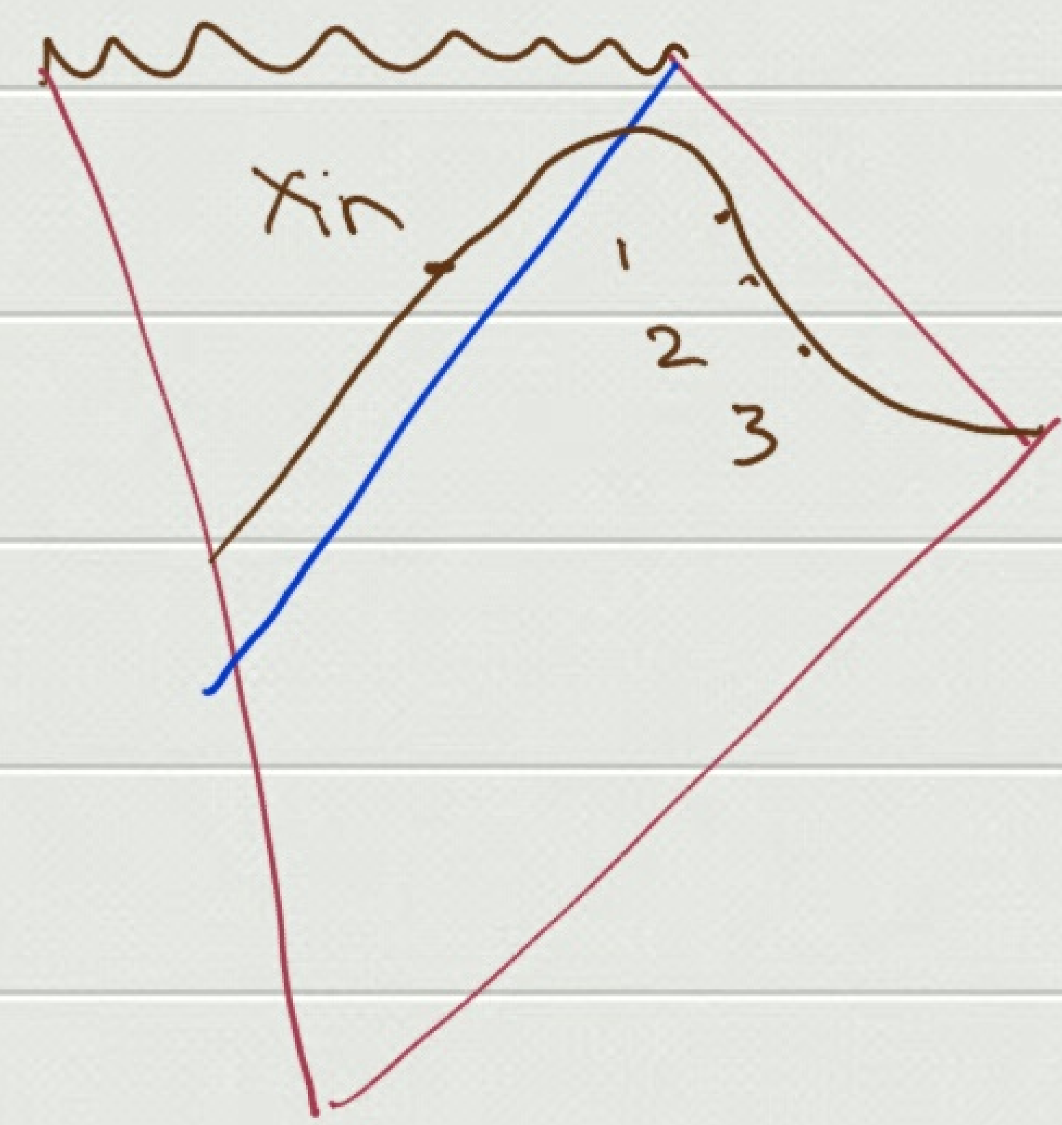
right at the complementarity bound.

Loss of locality in high-point correlators

In the previous approach to complementarity, attention was not paid to the question of how one could read information in the Hawking radiation using local observations.

The approach to complementarity we now describe states that inherently the space outside and inside are not independent.

More precisely, on a nice slice cloning is resolved by stating that



$$\phi(x_{in}) = P\left(\phi(x^{out}_1), \phi(x^{out}_2) \dots \phi(x^{out}_s)\right)$$

Notice that such a relation would be obviously wrong in a simple scalar theory.

There $\phi(x_i)$ at different points x_i are inherently distinct degrees of freedom. We have

$$[\phi(x_i), \phi(x_j)] = 0$$

if x_i, x_j are on the same spacelike slice and $x_i \neq x_j$. So obviously a relation of the form above cannot be true.

On the other hand if such a relation holds, then, if $\Pi(x_{in})$ is the canonical momentum we have

$$[\Pi(x_{in}), P(\phi(x_1^{out}), \ldots \phi(x_S^{out}))] = 1.$$

But this large commutator (directly linked to the non-factorization of the H-space) is visible only in high-point correlators

It is invisible in low-point correlators.

Next time we will explicitly see an example of this kind in AdS/CFT.

But the roots of this phenomenon appear to lie deep within gauge theories.

First, we describe this using a simple example from QED and then we generalize to gravity

Let us write the Lagrangian as

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J_\mu A^\mu + L_{matter}$$

Here the fields are A_μ and the canonical momenta are

$$\pi^\mu = -F^{0\mu}$$

The Poisson brackets, by definition are

$$[A_\mu(x), \pi^\nu(x')] = \delta^\nu_\mu \delta^{d-1}(x-x').$$

But to quantize the theory we have to deal with the constraints. We get a primary constraint

$$\pi^0 = 0$$

and a secondary first class constraint

$$\partial_i \pi^i + J^0 = 0 \quad \text{[Gauss law]}$$

We can promote these constraints to second class by fixing gauge.

But then during quantization, we have to promote Dirac brackets to commutators, not Poisson brackets.

For example in the gauge $A_z = 0$, we get commutators

$$[\Pi^z(t, x_1), \phi(t, x_2)] = i\, \theta(z_1 - z_2)\, \delta^{d-1}(x_1 - x_2)$$

which is manifestly non-local.

In gauge invariant language, we should instead consider the gauge invariant operator

$$\phi_{GI}(x) = P\left\{ e^{i\int A^\mu(x)\, dx_\mu} \right\} \phi(x)$$

which is manifestly non-local.

Precisely the same phenomenon appears in gravity. Recall

$$H = \frac{1}{16\pi} \int (\partial_i h_{ij} - \partial_j h_{ii}) \cdot n^j \, dA .$$

for a sphere at ∞ with normal n^j. [i,j run only over spatial indices.]

If we fix some specific gauge, operators like $\phi(x)$ appear local but in reality they satisfy

$$\frac{d\phi}{dt} = i [H, \phi]$$

which clearly indicates non-local commutators.